14. *Surjectivity for a Class of Dissipative Operators*

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(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 12, 1983)

Let X be a real Banach space with the norm $\|\cdot\|$. We will call a mapping $A: X\to 2^X$ a (multi-valued) operator in X. The domain of A is the set $D(A)=\{x\in X;\ Ax\neq\phi\}$ and the range of A is the set $R(A)=\bigcup_{x\in D(A)} Ax$. We also define the generalized domain $D_a(A)$ of A as the set of $x\in X$, for which there exist sequences x_n, y_n and a positive number M such that $x_n\to x$ as $n\to\infty$, $y_n\in Ax_n$ and $\|y_n\|\leqq M$ for $n=1,2,\cdots$. We denote the infimum of such numbers M by $|Ax|$ for each $x\in D_a(A)$. It is easy to show that $D(A)\subset D_a(A)\subset \overline{D(A)}$ and the set $\{x\in D_a(A);\ |Ax|\leqq r\}$ is closed in X for each $r\geqq 0$.

For operators A and B in X and scalars λ and μ, we define the operator $\lambda A+\mu B$ by setting $(\lambda A+\mu B)x=\{\lambda y+\mu z;\ y\in Ax \text{ and } z\in Ax\}$ for $x\in D(A)\cap D(B)$ and $D(\lambda A+\mu B)=D(A)\cap D(B)$. An operator A in X is single-valued if Ax is singleton for $x\in D(A)$ and, in this case, we use Ax to denote both the singleton and its element. Let I denote the identity operator in X.

An operator A in X is said to be dissipative if

$$\|x_1-x_2\|\leqq\|x_1-x_2-\lambda(y_1-y_2)\|$$

for $\lambda>0$, $x_i\in D(A)$ and $y_i\in Ax_i$, $i=1,2$. A dissipative operator A in X is said to be m-dissipative if $R(I-\lambda A)=X$ for $\lambda>0$.

The following theorem is proved in [4], Corollary 2.

Theorem 1. *Let A be a dissipative operator in X. Then the following* (i) *and* (ii) *are equivalent to each other.*

(i) *A is m-dissipative.*

(ii) *For each $x\in D_a(A)$ and $w\in X$, there exists a sequence $\delta_n>0$ such that $\delta_n\to 0$ as $n\to\infty$ and $R(I-\delta_n(A+w))\ni x$ for $n=1,2,\cdots$.*

As an application of this result, the following theorem due to Browder [1] can be readily proved.

Theorem 2. *Let A be a single-valued dissipative operator in X which is locally Lipshitz continuous on $X=D(A)$. Then A is m-dissipative.*

Some other applications of Theorem 1 can be found in [4]. Recently, in [5], Ray gave a simple proof of above Theorem 2, by using Caristi's fixed point theorem established in [2]. The purpose of this paper is to show that the Ray's argument also applies to the proof of Theorem 1. In fact, we establish the following general lemma, from