120. *Representation of the Generator and the Boundary Condition for Semigroups of Operators of Kernel Type*

By Tadashi UENO
College of General Education, University of Tokyo

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1. Let $P(t, x, E)$ be a Markov transition probability on a compact manifold D such that $\{P_t, t\geq 0\}$, given by

$$P_t f(x)=\int_D f(y)P(t, x, dy),$$

is a semigroup on $C(D)$. Then, it is known that, under a certain regularity condition, the generator A of $\{P_t, t\geq 0\}$ is represented as a second order integro-differential operator for smooth f in the domain of A([5]). This type of theorems originally go back to Kolmogorov [3], and various versions are obtained as in Yosida [7] and others.

If D is a bounded open domain with smooth boundary in a manifold, and if $\{P_t, t\geq 0\}$ is a diffusion semigroup on $C(\bar{D})$, then smooth functions in the domain of the generator satisfy a boundary condition given by a second order integro-differential operator under a certain regularity condition. This was obtained by Wentzell [6] as a partial extension of Feller [1], [2] for one dimensional diffusion.

Here, in this note, we extend the representation theorems of this type for a complex valued kernel $Q(t, x, E)$. The point is that $Q(t, x, E)$ has not the non-negative property, and the orders of the corresponding integro-differential operators are no more bounded by 2. They depend essentially on the order of $|Q|(t, x, E)$ near the point x as $t\searrow 0$, where $|Q|$ is the measure given by the variation of Q. Neither the semigroup property nor the regularity of

$$Q_t f(x)=\int_D f(y)Q(t, x, dy),$$

as a function of x, are essential for the representations. But, the corresponding propositions for semigroups can be derived easily from Theorems 1–4. The proofs of theorems will be published elsewhere.

2. Let D be a manifold, or an open domain with boundary ∂D in a manifold of dimension N, where the manifold and ∂D are of class C^∞. For a fixed point x in $\bar{D}=D\cup\partial D$,[1)] let $\{\xi_k^{(x)}(y), 1\leq k\leq N\}$ be a local coordinate in a neighbourhood of x, such that $\xi_k^{(x)}(y)$'s are defined and continuous on $\bar{D}$, and $\xi_k^{(x)}(y)=0$, $1\leq k\leq N$, if and only if $y=x$. When

1) When D is a manifold, we understand that $\partial D=\phi$ and $\bar{D}=D$.