118. On Rational Similarity Solutions of KdV and m-KdV Equations

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§1. **Summary.** The Korteweg-de Vries (KdV) equation

(1.1) $$u_t-12uu_x+u_{xxx}=0$$

and the modified Korteweg-de Vries (m-KdV) equation

(1.2) $$v_t-6v^2v_x+v_{xxx}=0$$

have a series of rational similarity solutions

(1.3) $$u_n(x,t)=g(n+1)x^{-2}-\left(\frac{\partial}{\partial x}\right)^2\log F_{n+1}(x,t),$$

(1.4) $$v_n(x,t)=(g(n)-g(n+1))x^{-1}+\frac{\partial}{\partial x}\log(F_n(x,t)/F_{n+1}(x,t))$$

where

(1.5) $$F_n(x,t)=\sum_{j=0}^{f(n)}P_{n,j}(3t)^j x^{3(f(n)-j)}$$

is a homogeneous polynomial of x^3 and t of degree $f(n)=[n(n-1)/6]$ with integral coefficients $P_{n,j}$ ($P_{n,0}=1$, $P_{n,f(n)}\neq 0$), $g(n)=1$ if $n\equiv 2$ (mod 3), $=0$ otherwise. These polynomials are essentially the same as those of A. I. Yablonskii [1] and A. P. Vorobiev [2]. Actually the polynomials

(1.6) $$P_n(\xi)=\sum_{j=0}^{f(n)}P_{n,j}\xi^{d(n)-3j},\qquad (d(n)=n(n-1)/2)$$

were introduced by them to describe the rational solutions of Painlevé-II equation.

(1.7) $$q_n=(\log P_n(\xi)/P_{n+1}(\xi))'$$

satisfies Painlevé-II equation

(1.8) $$q_n''=2q_n^3+\xi q_n+n.$$

It gives also a rational solution of the Toda equation. If p_n is given by

(1.9) $$p_n=-P_nP_{n+2}/4P_{n+1}^2=(\log P_{n+1}(\xi))''-\xi/4$$

then $\{q_n, p_n\}$ satisfies the Toda equation

(1.10) $$q_n'=p_{n-1}-p_n,\qquad p_n'=p_n(q_n-q_{n+1}).$$

Vorobiev calculated the coefficients of P_n ($n\leq 8$) and showed that $P_{n,j}$ are very large integers for large n and j. Here we give a theoretical bound for them.

(1.11) $$|P_{n,j}|\leq(7n)^{4j},\quad n=1,2,3,\cdots,\quad 0\leq j\leq f(n).$$