115. The Gauss Map in Models

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1. Introduction. Let N be an n-dimensional Riemannian manifold isometrically immersed into a Euclidean $(n+k)$-space E^{n+k} $(k\geqq 1)$ and $\mathcal{V}_E(N)$ be the unit normal bundle of N in E^{n+k}. Then the Gauss map of $\mathcal{V}_E(N)$ into the unit sphere about the origin of E^{n+k} was given by Chern and Lashof [1]. J. L. Weiner [5] gave a generalization of this map as follows: Let N be an isometrically immersed n-dimensional Riemannian manifold into a complete $(n+k)$-dimensional Riemannian manifold. Suppose that for a point p of N, N does not intersect the cut locus of p. The parallel displacement of $v\in\mathcal{V}_M(N)$ (=the unit normal bundle of N in M) along the shortest geodesic segment joining the foot point of v to p gives a mapping of $\mathcal{V}_M(N)$ into the unit sphere in the tangent space of M at p. This map is called the Gauss map on N based at p. R. Takagi [4] described an n-dimensional complete Riemannian N isometrically immersed into a Euclidean $(n+1)$-sphere S^{n+1} when the Gauss map on N based at a point S^{n+1} has constant rank. Furthermore, J. L. Weiner [5] showed similar results when the ambient space is a hyperbolic space of curvature -1 and also reproved Takagi's theorem in a simpler fashion. When the ambient space M is a model with a pole o, the cut locus of o is empty. So, for any isometrically immersed Riemannian manifold N into M, the Gauss map G_M on N based at o can be defined. In this note, we will study the Gauss map G_M and show the similar results to those of J. L. Weiner.

2. Preliminaries. Let (M, o) be an n-dimensional model with a pole o $(n\geqq 2)$ and $h:=\mathrm{Exp}_o: M_o\to M$ be the exponential map from the tangent space M_o at o of M onto M. Choosing an orthonormal basis $\{e_1, \cdots, e_n\}$ on M_o, let $\{y^1, \cdots, y^n\}$ be the normal coordinate system relative to this basis. Let g be the Riemannian metric on M. Then h^*g is a Riemannian metric on M_o and written by

$$h^*g=dr^2+f(r)^2d\Theta^2.$$

Here $d\Theta^2$ denotes the canonical metric on the unit sphere of M_o, r is the usual radial function on M_o and $f(r)$ is the C^∞ function on $[0,\infty)$ satisfying

$$f(0)=0,\ f'(0)=1,\ f(r)>0 \qquad \text{for } r>0.$$

3. Parallel displacements. For a tangent vector