112. *On Certain Cubic Fields. IV*

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1. We shall use the following notations: For an algebraic number field k, the discriminant, the class number, the ring of integers and the group of units are denoted by $D(k)$, $h(k)$, $\mathcal{O}_k$ and E_k respectively. The discriminant of an algebraic integer ρ will be denoted by $D_k(\rho)$ and the discriminant of a polynomial $h(x) \in \boldsymbol{Z}(x)$ by D_h. $(*/*)$ means the quadratic residue symbol.

The purpose of this note is to give some devices of generating cubic fields of certain types with even class numbers. We shall prove:

Theorem A. *Let* $K=\boldsymbol{Q}(\theta)$, $\mathrm{Irr}\,(\theta\,;\boldsymbol{Q})=f(x)=x^3-mx^2-(m+3)x-1$, $m \in \boldsymbol{Z}$ *with odd* m *and* $m \geqq 1$. *Suppose there exists a prime number* q *satisfying*

(i) $(r(\theta), q)=1$, *where* $D_K(\theta)=r(\theta)^2 D(K)$,

(ii) $f(x)\equiv(x+a)(x+b)(x+c) \pmod q$, *where any two of* $a, b, c \in \boldsymbol{Z}$ *are not congruent* mod q, $a>0$, $a\not\equiv 0, m, m+1 \pmod 4$,

(iii) $((a-b)/q)=-1$,

(iv) $-f(-a)=a^3+ma^2-(m+3)a+1=t^2$ *for some odd* $t \in \boldsymbol{Z}$.

Then we have $2\,|\,h(K)$.

Theorem A′. *Let* $K=\boldsymbol{Q}(\theta)$, $\mathrm{Irr}\,(\theta\,;\boldsymbol{Q})=f(x)=x^3-mx^2-(m+3)x-1$, $m \in \boldsymbol{Z}$ *with* $3\nmid m$ *and* $m\geqq 1$.

(I) *Suppose* $m\equiv 3 \pmod 4$ *and* $2m+3=u^2$ *for some* $u \in \boldsymbol{Z}$. *If* $2m+3$ *has a prime factor* q *such that* $q=12s\pm 5$, *then we have* $2\,|\,h(K)$. *Examples*: 11, 23. *It is easy to see that there are infinitely many m's satisfying this condition.*

(II) *Suppose* $m\equiv 1 \pmod 4$. *Let* q *be a prime factor* $(\neq 7)$ *of* $6m+19$. *Then we have*

$(*)$ $f(x)\equiv(x+3)(x+b)(x+c) \pmod q$, *where* $b\not\equiv 3$, $c\not\equiv 3 \pmod q$.

If $6m+19=v^2$ *for some* $v \in \boldsymbol{Z}$ *and* $((3-b)/q)=-1$ *in* $(*)$, *we have* $2\,|\,h(K)$. *Examples*: $m=17, 25$.

Theorem B. *Let* $F=\boldsymbol{Q}(\delta)$, $\mathrm{Irr}\,(\delta\,;\boldsymbol{Q})=g(x)=x^3-nx^2-(n+1)x-1$, $u \in \boldsymbol{Z}$ *with* $n\equiv 3 \pmod 4$ *but* $n\neq 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, -6$. *If* D_g *is square free, then we have* $2\,|\,h(F)$. *Examples*: $n=7, 11, 15$.

2. *Proof of Theorem* A. As $\sqrt{D_f}=m^2+3m+9 \in \boldsymbol{Z}$, $K/\boldsymbol{Q}$ is totally real and Galois. In virtue of (i), (ii), (q) is completely decomposed in K in the form $(q)=\mathfrak{q}_1\mathfrak{q}_2\mathfrak{q}_3$, where $\mathfrak{q}_1=(q, \theta+a)$, $\mathfrak{q}_2=(q, \theta+b)$,