111. *Signature of Quaternionic Kaehler Manifolds**)

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We announce that the signature (or the index) of a compact quaternionic Kaehler manifold M equals the Betti number $b_{2n}(M)$, $\dim M = 4n$, and state properties of its cohomology (Theorem 2.6).

1. Facts from Salamon's work. (1.1) **Definition.** A manifold with Riemannian metric (M, g) is called *quaternionic Kaehlerian* iff the linear holonomy group Ψ_x is contained in $Sp(n)\cdot Sp(1)\subset O(TM_x, g_x)$ for every point x of M, $\dim M=4n$, $n\geqq 2$.

Corresponding to the Lie algebra of $Sp(1)$, there is a parallel vector subbundle V of End $(TM)=TM\otimes T^*M$. (V is a coefficient bundle of imaginary quaternions in [4].) Let Z denote the submanifold of V which consists of the members J satisfying $J^2=-$(the identity map of TM_x), $x=\pi(J)=\text{proj}\,(J)$. Then Z is a sphere bundle $S^2 \longrightarrow Z \overset{\pi}{\longrightarrow} M$. (See [4].)

Now we construct an almost complex structure on Z which is known to be integrable [4]. Observe that Z is a parallel fibre subbundle of V and each fibre S^2 has a natural complex structure. Furthermore the tangent space TZ_J to Z at every point J is the direct sum of the tangent space to the fibre and the horizontal space which is isomorphic by the projection with the tangent space TM_x, $x=\pi(J)$, with the complex structure J. Thus one has a complex structure on TZ_J in the obvious fashion.

(1.2) **Definition.** The complex manifold Z with the projection π is the *twistor space* of the quaternionic Kaehler manifold (M, g).

(1.3) **Hypothesis.** We assume that (M, g) *is a compact connected quaternionic Kaehler manifold of positive scalar curvature.*

(1.4) **Theorem** (Salamon [4]). *Under the hypothesis* (1.3), *the twistor space Z has a unique Kaehler metric such that* (1) *the projection π is a Riemannian submersion,* (2) *the aforementioned horizontal subspace is orthogonal to the fibre, and* (3) *the metric on the fibre of Z is a constant multiple of the metric induced from g on M at every point of Z.*

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