110. A Note on Γ-Rings

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Introduction. Throughout the paper, M stands for a Γ-ring, as defined by Barnes [1]. We shall utilize the standard notations and definitions in Barnes [1] and Hsu [2]. In [2] Hsu has introduced the notion of g-prime ideals in Γ-rings and proved that for any ideal A of the Γ-ring M, the radical $r_g(A)$ of A (that is, the set of all elements x of M such that every g-system containing x contains an element of A) is the intersection of all g-prime ideals containing A. In this paper we introduce the notion of g-halfprime ideals in Γ-rings and prove that an ideal A of the Γ-ring M is g-halfprime if and only if $A=r_g(A)$.

Preliminary definitions. If a is an element of the Γ-ring M, then $\langle a\rangle$ denotes the principal ideal generated by a. If S is a subset of M, we call S an *sp-system* if $S=\emptyset$ or $a\in S$ implies $\langle a\rangle^2\cap S\neq\emptyset$. A non-empty subset S of M is called a *g-sp-system* if S contains an sp-system S' such that $g(x)\cap S'\neq\emptyset$ for every element x of S, where S' is called a *kernel* of S. An ideal I of M is said to be *g-halfprime* if $C(I)=M\backslash I$ is a g-sp-system.

Example. Consider Z, the ring of integers, as a Γ-ring with $\Gamma=Z$. Let p, q be two distinct prime numbers. Define $g(a)=\langle\{a, pq\}\rangle$. Now $g(pq)=\langle pq\rangle$ and hence $C(\langle pq\rangle)$ is g-sp-system with kernel $C(\langle pq\rangle)$, which is not a g-system.

Suppose K is a subset of M and satisfies the condition: For each $a\in K$, there exists an sp-system $S\subseteq K$ such that $g(a)\cap S\neq\emptyset$. Then consider the set X, which is the union of all sp-systems which are contained in K. One can easily verify that K is a g-sp-system with kernel X. Hence a subset K of M is a g-sp-system if and only if K satisfies the condition: For each $a\in K$, there exists an sp-system $S\subseteq K$, such that $g(a)\cap S\neq\emptyset$.

Main Theorem. Before proving our main theorem, we prove the following

Lemma. *If S is an sp-system and $x\in S$, then there exists an m-system X* (Def. 3.2. in [2]) *such that $x\in X$ and $X\subseteq S$.*

Poof. Let S be an sp-system and x an element of S. Then there exists an element $x_1\in\langle x\rangle^2\cap S$. Again since S is an sp-system, there exists $x_2\in\langle x_1\rangle^2\cap S$. If we continue this process, we get a sequence $\{x_i\}$ of elements in S with $x_0=x$ and $x_{i+1}\in\langle x_i\rangle^2\cap S$ for $i\geq 0$. Now x_i