109. On a Question Posed by Huckaba-Papick. II

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1. Introduction. This is a continuation of [5]. As in the introduction of [5], let R be an integral domain with the quotient field K, and let x be an indeterminate. By $c(f)$ we denote the ideal of R generated by the coefficients of f for an element f of $R[x]$. We denote the subset $\{f \in R[x]; c(f)^{-1}=R\}$ of $R[x]$ by U, where $c(f)^{-1}=\{a \in K; ac(f) \subset R\}$. Let $\mathcal{P}(R)$ be the set of prime ideals of R which are minimal prime ideals over $(a: b)$ for some elements a, b of R. Huckaba-Papick ([2]) posed the following questions:

Questions ([2, Remark (3.4)]). (a) If R_P is a valuation ring for each $P \in \mathcal{P}(R)$, is $R[x]_U$ a Prüfer ring?

(b-1) If $R[x]_U$ is a Bezout ring, are the prime ideals of $R[x]_U$ extended from prime ideals of R?

(b-2) If $R[x]_U$ is a Prüfer ring, are the prime ideals of $R[x]_U$ extended from prime ideals of R?

(c) If $R[x]_U$ is a Prüfer ring, is it a Bezout ring?

In [4], we answered to the question (b-1) in the affirmative, and showed that questions (b-2) and (c) are equivalent. In [5], we answered to the question (c) in the affirmative. The purpose of this paper is to give a negative answer to the question (a) in proving the following result:

Proposition. *There exists an integral domain R such that R_P is a valuation ring for each $P \in \mathcal{P}(R)$ and that $R[x]_U$ is not a Prüfer ring.*

2. Proof of Proposition. **Lemma 1.** *If $R[x]_U$ is a Prüfer ring, then the prime ideals of $R[x]_U$ are extended from prime ideals of R.*

Proof. By [5, Theorem 1], $R[x]_U$ is a Bezout ring. By [4, Theorem 1], the prime ideals of $R[x]_U$ are extended from prime ideals of R.

Throughout the rest of the paper, we denote by R the integral domain $Z[2u, 2u^2, 2u^3, \cdots]$ where u is an indeterminate over Z, and by K the quotient field of R (cf. [1, §25, Exercise 21]).

Lemma 2 ([3, II, a part of Example 2]). (1) *The maximal ideal $M=(2, 2u, 2u^2, \cdots)$ of R is a minimal prime ideal over the principal ideal* (2).

(2) *R_M is a valuation ring.*

(3) *M is the only maximal ideal of R containing* 2.

(4) *R is integrally closed.*