108. An Approach by Difference to a Quasi-Linear Parabolic Equation

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1. Introduction. This paper treats the semi-group associated with the Cauchy problem for the equation

(1) $\partial u/\partial t=\Delta\phi(u)$ for $t>0$ and $x\in R^N$ $\left(\Delta=\sum_{i=1}^{N}\partial^2/\partial x_i^2\right)$

through the difference scheme

(2) $$h^{-1}(u(t+h,x)-u) =\sum_{i=1}^{N}L^{-2}\{\phi(u(t,x+Le_i))-2\phi(u)+\phi(u(t,x-Le_i))\},\quad (e_i=(0,\cdots,0,\overset{i}{1},0,\cdots,0))$$

where ϕ is a differentiable function on R with $\phi(0)=0$ such that ϕ' is non-negative and bounded on every finite sub-interval of R. The convention

(3) $$C_i(t)f(x)=(f(x+te_i)+f(x-te_i))/2 \qquad (i=1,\cdots,N)$$

enables us to rewrite (2) as

(2)′ $$h^{-1}(u(t+h,x)-u(t,x))=\sum_{i=1}^{N}2L^{-2}(C_i(L)-I)\phi(u(t,x)),$$

and provides a strongly continuous cosine family $C_i(t)$, $t\in R$ in a Banach space $L^1(R^N)$ with norm $\|\cdot\|_1$ for each fixed i. For cosine families in Banach spaces, see [7] for example.

The Cauchy problem for (1) arises in mathematical models of many physical situations. The semi-group approaches to (1) in $L^1(R^N)$ were made by Benilan, Brezis and Crandall (see [2], [4]). The method is essentially based on their theory on the semi-linear equation $\phi^{-1}(u)-\Delta u=f$ developed in [1]. But, our method is more constructive and provides applications to numerical analysis for (1). Indeed, our main task is to show that

$$\left(I-\lambda\sum_{i=1}^{N}2L^{-2}(C_i(L)-I)\phi\right)^{-1} \quad \text{converges in } L^1(R^N) \text{ as } L\downarrow 0.$$

2. Main results. Consider the operator C_h defined by

(4) $$C_hu=u+h\sum_{i=1}^{N}2L^{-2}(C_i(L)-I)\phi(u),$$

where $h, L>0$ and $L^2=2Nh\sup_{|r|\leq m}\phi'(r)$ for an integer m. Let A_i be, for each i, the infinitesimal generator of the strongly continuous cosine family $C_i(t)$, $t\in R$ in $L^1(R^N)$ defined by (3), and let $\overline{A}$ be the smallest closed extension of $\sum_{i=1}^{N}A_i$ in $L^1(R^N)$. We are concerned with a gener-