12. *Hodge Filtrations on Gauss-Manin Systems. II*

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(Communicated by Kunihiko KODAIRA, M. J. A., Feb. 12, 1983)

Let $f: X \to Y$ be a projective morphism of algebraic manifolds. The theory of Deligne, Gabber, Beilinson, and Bernstein describes the decomposition of the direct image $\boldsymbol{R}f_*\boldsymbol{C}_X$ in $D_c^b(\boldsymbol{C}_Y)$, and gives the Poincaré duality and the hard Lefschetz theorem (cf. [3]). We prove the theorem for a one-parameter projective family (i.e., f is flat projective and $\dim Y=1$) without assuming algebraicity (cf. Theorem (1.1) and Corollary (1.2)). We use essentially the theory of filtered $\mathcal{D}$-Modules [1], [5], which enables us to apply the theory of limit mixed Hodge structure of Steenbrink.

I would like to thank Profs. K. Saito and M. Kashiwara for useful discussions.

§ 1. Let $f: Y \to S$ be a projective morphism of complex manifolds with $\dim Y=n+1$ and $\dim S=1$. In [5], we defined the Gauss-Manin system $\int_f \mathcal{O}_Y$ in $DF(\mathcal{D}_S)$, such that $DR_S\left(\int_f \mathcal{O}_Y\right) \simeq \boldsymbol{R}f_*\boldsymbol{C}_Y$ in $D_c^b(\boldsymbol{C}_S)$ (cf. [1], [4]).

(1.1) **Theorem.** 1) *We have the isomorphisms*

(1.1.1) $$\int_f \mathcal{O}_Y \simeq \sum_k \int_f^k \mathcal{O}_Y[-k] \qquad \textit{in } DF(\mathcal{D}_S),$$

and

(1.1.2) $$\int_f^k \mathcal{O}_Y \simeq \mathcal{H}_\Sigma^0\left(\int_f^k \mathcal{O}_Y\right) \oplus {}^\pi\left(\int_f^k \mathcal{O}_Y|_{S^*}\right) \qquad \textit{for any } k \in \boldsymbol{Z},$$

as filtered $\mathcal{D}_S$*-Modules. Here,*

$$\int_f^k \mathcal{O}_Y = \mathcal{H}^k\left(\int_f \mathcal{O}_Y\right),$$

$\Sigma = S - S^*$ *is the set of the critical values of* f *and* ${}^\pi\mathcal{M}$ *is the minimal extension of a regular holonomic system* $\mathcal{M}$ *on* S^* (i.e., $\mathcal{H}_\Sigma^0({}^\pi\mathcal{M}) = \mathcal{H}_\Sigma^0({}^\pi\mathcal{M}^*)=0$ *and* ${}^\pi\mathcal{M}|_{S^*} \simeq \mathcal{M}$).

2) *Let* L *be a relatively ample line bundle on* Y *and let us also denote by* L *the operator defined by the cup product of* $c_1(L)$. *Then we have the isomorphisms for any* $k \in \boldsymbol{Z}_+$

(1.1.3) $$L^k: \int_f^{n-k} \mathcal{O}_Y \xrightarrow{\sim} \int_f^{n+k} \mathcal{O}_Y\{k\},$$

and

(1.1.4) $$\int_f^{n-k} \mathcal{O}_Y \simeq \left(\int_f^{n+k} \mathcal{O}_Y\right)^* \{-n\},$$