105. *Boundedness of Singular Integral Operators of Calderón Type*

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§1. **Introduction.** Let $K(x,y)$ be a kernel satisfying $|K(x,y)| \leqq \mathrm{Const.}/|x-y|$ for any pair (x,y) of real numbers with $x \neq y$. We say that $K(x,y)$ is of type 2 if $Kf(x)=\lim_{\varepsilon\to 0}\int_{\varepsilon<|x-y|<1/\varepsilon} K(x,y)f(y)\,dy$ exists almost everywhere for any $f \in L^2$ and $\|K\|_2=\sup\{\|Kf\|_2/\|f\|_2;\ f\in L^2\}<\infty$, where L^2 denotes the space of square integrable functions $f(x)$ on the real line with norm $\|f\|_2=\left\{\int_{-\infty}^{\infty}|f(x)|^2dx\right\}^{1/2}$. For the harmonic analysis on curves, A. Calderón investigated kernels $C[\phi](x,y) =1/\{(x-y)+i(\phi(x)-\phi(y))\}$ for real-valued functions $\phi(x)$ and, in [2], he showed that $C[\phi]$ is of type 2 as long as $\|\phi'\|_\infty=\mathrm{ess.\,sup}_x|\phi'(x)|$ is sufficiently small. Using this theorem he also studied kernels

$$(1)\qquad C[h,\phi](x,y)=\frac{1}{x-y}\,h\left\{\frac{\phi(x)-\phi(y)}{x-y}\right\}$$

for complex-valued functions $h(t)$ and real-valued functions $\phi(x)$. In [5], R. Coifman-A. McIntosh-Y. Meyer showed that $C[\phi]$ is of type 2 if $\|\phi'\|_\infty<\infty$. Using this theorem, R. Coifman-G. David-Y. Meyer showed, in [4], the following

Theorem. *If $h(t)$ is infinitely differentiable, then $C[h,\phi]$ is of type 2 as long as $\|\phi'\|_\infty<\infty$.*

The purpose of this paper is to give a new proof of this theorem. We shall deduce this theorem from Calderón's theorem and so-called "good λ inequalities". The author expresses his thanks to Prof. A. Uchiyama, through whose notebook the author learned recent Calderón's lecture on $C[\phi]$.

§2. **Proof of Theorem.** Without loss of generality we may assume that $h(t)$ has a compact support. Let $\hat{h}(\xi)=\int_{-\infty}^{\infty}e^{-i\xi t}h(t)\,dt$. Then we have formally

$$(2)\qquad C[h,\phi](x,y)=\mathrm{Const.}\int_{-\infty}^{\infty}\hat{h}(\xi)C[e^{i\xi\cdot},\phi](x,y)\,d\xi,$$

and hence it is natural to investigate kernels $K[\psi]=C[e^{i\cdot},\psi]$ for real-valued functions $\psi(x)$. For a locally integrable function $f(x)$, we put