100. Microlocal Study of Sheaves. I
Contact Transformations

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0. **Introduction.** In [2] we defined the micro-support of a complex of sheaves F on a real manifold X and studied its functorial properties. With this tool, we are now able to quantize contact transformations for any sheaves. We prove that such q.c.t. commute with the Sato microlocalization, and when the manifolds are complex analytic we prove that the structure sheaf $\mathcal{O}_X$ is invariant by q.c.t.

1. Let X be a real manifold of class C^α $(2\leq\alpha\leq\infty$, or $\alpha=\omega)$, T^*X its cotangent bundle, and π the projection from T^*X to X. Let A be a commutative ring. We denote by $D^+(X)$ (resp. $D^b(X)$) the full subcategory of the derived category of complexes of sheaves of A-modules on X whose cohomology is bounded from below (resp. bounded).

Let $F\in Ob(D^+(X))$. The micro-support of F, $SS(F)$, is a closed conic subset of T^*X defined in [2]. Let Ω be a subset of T^*X. We set:

$$\mathcal{E}(\Omega)=\{F\in Ob(D^+(X))\,;\, SS(F)\cap\Omega=\emptyset\}.$$

Let $S(\Omega)$ be the set of morphisms in $D^+(X)$, $u: F\to G$, such that the mapping cone of u belongs to $\mathcal{E}(\Omega)$. Then $S(\Omega)$ satisfies the axioms of [1], which enable us to localize $D^+(X)$ by $S(\Omega)$. We denote by $D^+(X,\Omega)$ the triangulated category so constructed (for $p\in T^*X$ we write $D^+(X,p)$ instead of $D^+(X,\{p\})$).

Let q_j be the j-th projection from $X\times X$, $(j=1,2)$ and let Δ be the diagonal of $X\times X$. For F and $G\in Ob(D^+(X))$, we define:

$$\mu\hom(F,G)=\mu_\Delta(\boldsymbol{R}\mathcal{H}om(q_2^{-1}F,q_1^!F)).$$

Recall that for a submanifold $Y\subset X$, $\mu_Y(*)$ is the functor of the Sato microlocalization along Y([4]). Thus $\mu\hom(F,G)$ is a complex of sheaves on $T^*X\simeq T^*_\Delta(X\times X)$,

Proposition 1. *Let $p\in T^*X$. Then there exists a natural isomorphism:*

$$\mathrm{Hom}_{D^+(X,p)}(F,G)\simeq\mathcal{H}^0(\mu\hom(F,G))_p.$$

2. Let (E,σ) be a real symplectic vector space, and λ_1, λ_2, λ_3 three Lagrangian planes in E. Let q be the quadratic form on $\lambda_1\oplus\lambda_2\oplus\lambda_3$ given by:

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