98. *The Structure of Serial Rings and Self-Duality*

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The notion of serial rings was introduced by T. Nakayama [6]. A left and right Artinian ring R is called *serial* if Re as well as eR has a unique composition series for any primitive idempotent $e \in R$. The structure of serial rings has been studied by many authors (cf. [1], [3]). The purpose of this note is to give a method for the construction of serial rings in general, and we shall give a necessary and sufficient condition for given two serial rings to be Morita equivalent to each other. Moreover, as one of its applications, we shall prove that a serial ring satisfying a mild condition has a self-duality. Proofs and details will be published elsewhere.

1. Let $b_1, b_2, \cdots, b_n$ be a sequence of positive integers such that $b_i \geqq 2$ for $i=2, 3, \cdots, n$ and $b_{[i+1]} \leqq b_i+1$ for $i=1, 2, \cdots, n$, where $[k]$ denotes the least positive remainder of k modulo n. For each i, let us put $c_i=(1/n)\{b_i-[b_i]\}+1$ and $d_i=(1/n)\{b_{[i+1]}-1-[b_{[i+1]}-1]\}+1$. Let $R_1, R_2, \cdots, R_n$ be local uniserial rings such that $c({}_{R_i}R_i)=c_i$ and $R_i/(J_i)^{d_i} \cong R_{[i+1]}/(J_{[i+1]})^{d_i}$ for all i, where $J_i=\mathrm{Rad}\,(R_i)$ and $c(M)$ denotes the composition length of a module M. Let $\varphi_i: R_i \rightarrow R_{[i+1]}$ be a function and $w_i \in R_i$, $i=1, 2, \cdots, n$. Then the system $\mathfrak{S}=\{n; b_i, R_i, w_i, \varphi_i\}$ is called a *serial system* if the following four conditions are satisfied: For each i,

(i) $J_i=R_iw_i=w_iR_i$,

(ii) $\pi_{[i+1]} \circ \varphi_i$ is an onto ring homomorphism where $\pi_{[i+1]}: R_{[i+1]} \rightarrow R_{[i+1]}/(J_{[i+1]})^{d_i}$ denotes the canonical ring homomorphism,

(iii) $\varphi_i(w_i) \equiv w_{[i+1]} \pmod{(J_{[i+1]})^{d_i}}$,

(iv) $r_iw_i=w_i\varphi_{[i-1]} \circ \varphi_{[i-2]} \circ \cdots \circ \varphi_i(r_i)$ for all $r_i \in R_i$.

Let R be an indecomposable self-basic serial ring with the radical J. Then we can construct a serial system $\mathfrak{S}_R$ associated to R, which will be called an *invariant system* of R, as follows: Let $Re_1, Re_2, \cdots, Re_n$ be a *Kupisch series* for R, i.e., $1_R=e_1+e_2+\cdots+e_n$ is a decomposition of 1_R into a sum of mutually orthogonal primitive idempotents such that $c({}_RRe_i) \geqq 2$ for $i=2, 3, \cdots, n$, $Je_i/J^2e_i \cong Re_{i-1}/Je_{i-1}$ for $i=2, 3, \cdots, n$, and $Je_1/J^2e_1 \cong Re_n/Je_n$ if $Je_1 \neq 0$. Let us put $b_i=c({}_RRe_i)$ and $R_i=e_iRe_i$, $i=1, 2, \cdots, n$. For each i, let y_i be an element in $e_iJe_{[i+1]}$ such that $e_iJe_{[i+1]}=R_iy_i=y_iR_{[i+1]}$, and define a function $\varphi_i: R_i \rightarrow R_{[i+1]}$