97. On v-Ideals in a VHC Order*)

By Hidetoshi MARUBAYASHI
College of General Education, Osaka University

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Throughout this note, Q will be a simple artinian ring and R will be an order in Q with 1. Let $\underline{C}$ ($\underline{C}'$) be a right (left) Gabriel topology on R cogenerated by the right (left) injective hull of Q/R. In [4], R is called a *VH (v-hereditary) order* if for any R-ideal A such that ${}_vA = A$ ($A_v = A$) we have ${}_v(A(R:A)_l) = O_l(A)$ (resp. $((R:A)_rA)_v = O_r(A)$). We say that R is a *VHC order* if it is a VH order satisfying the maximum condition on $\underline{C}$-closed right ideals and $\underline{C}'$-closed left ideals. The concept of VHC orders is a Krull type generalization of HNP (hereditary noetherian prime) rings. The aim of this note is to extend Robson's theorems and Fujita-Nishida's theorems in HNP rings to the case of VHC orders (cf. [1], [7] and [3]). Concerning our terminology and notations we refer to [4]. See [6] for many interesting examples of VHC orders.

Proposition 1. *The following two conditions are equivalent*:

(1) ${}_v(A(R:A)_l) = O_l(A)$ *for any R-ideal A such that* ${}_vA = A$.

(2) ${}_v(A(R:A)_l) = {}_v(O_l(A))$ *for any R-ideal A.*

Proof. (2)$\Rightarrow$(1) is clear, because ${}_v(O_l(A)) = O_l(A)$ for any R-ideal A with ${}_vA = A$. (1)$\Rightarrow$(2): Since ${}_vA \supset A$, we have $1 \in O_l({}_vA) = {}_v({}_vA(R:{}_vA)_l) \subset {}_v({}_vA(R:A)_l) = {}_v(A(R:A)_l)$ by Lemma 1.1 of [4]. It is clear that $A(R:A)_l \subset O_l(A)$ and so ${}_v(A(R:A)_l) \subset {}_v(O_l(A))$. On the other hand, $A(R:A)_l$ is an $(O_l(A), O_l(A))$-bimodule and thus ${}_v(A(R:A)_l)$ is a right $O_l(A)$-module. Hence it follows that $O_l(A) \subset {}_v(A(R:A)_l)$ and that ${}_v(O_l(A)) \subset {}_v(A(R:A)_l)$.

From now on, *R will be a VHC order in a simple artinian ring Q.*

Lemma 1. *Let A be any R-ideal. Then* ${}_vA = A_v$.

Proof. This is proved as in Lemma 1.2 of [4] by using Proposition 1.

We consider the following sets of v-ideals of R: $V(R) = \{A :$ ideal of $R \mid A : v\text{-ideal}\} \supset V_m(R) = \{A \in V(R) \mid A \subset P :$ prime v-ideal $\Rightarrow P :$ maximal v-ideal$\}$. If R has enough v-invertible ideals, then $V(R) = V_m(R)$ by Lemma 1.2 of [5]. We do not have an example of VHC order in which $V(R) \supsetneqq V_m(R)$ up to now. We study the properties of ideals belonging to $V_m(R)$.

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