96. *Some Dirichlet Series with Coefficients Related to Periods of Automorphic Eigenforms. II*[*)]

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§6. This paper is a direct continuation of [2]. Our primary objective here is to begin a discussion of several applications of the general formalism considered in §§2–5.

§7. We start by deriving some estimates for $F_\mu(\xi\,;S^{\pm 1})$. Cf. Theorem 2. The basic procedure is that of analytic number theory. By examining *appropriate* combinations of the Mellin transforms mentioned in [2, p. 416 (line 5)] and applying (4.1), we quickly establish that

$$|F_\mu(\xi\,;S^{\pm 1})|=O(1)e^{(\pi/2+\delta)|t|} \tag{7.1}$$

for $\xi=\sigma+it$, $|\sigma|\leqq N$, $|t|\geqq 1$, $\delta>0$. The implied constant may depend on N, ϕ, δ. Compare [6, pp. 311, 313] and [15, p. 22 (line 12)]. We (can) now combine a Phragmén-Lindelöf argument with (4.1) and theorem 2(v). Cf. [5, p. 95]. This yields:

Theorem 3. *Given* $0<\varepsilon<1/100$ *and* $N\geqq 3$. *Then*:

$$F_\mu(\xi\,;S^{\pm 1})=O\left[\frac{1}{\varepsilon}|t|^{\max(0,3/2-2\sigma,3/2+\varepsilon-\sigma)}\right]$$

for $\xi=\sigma+it$, $|\sigma|\leqq N$, $|t|\geqq 1$. *The implied constant depends solely on* (Γ, N, S, ϕ).

§8. Take $T\geqq 2x\geqq 2000$ and consider the integral

$$\frac{1}{2\pi i}\int_{\partial R}F_\mu(\xi\,;S)\frac{(2\pi x)^{\xi+1}}{\xi(\xi+1)}d\xi \qquad \text{for } \mu=a,\,b$$

with $R=[-\varepsilon,3/2+\varepsilon]\times[-T,T]$. Cf. [5, p. 31]. The "horizontal" contribution is easily estimated using Theorem 3. The contribution from $\{\sigma=-\varepsilon\}$ is then handled using Theorem 2(v) and [15, p. 62 middle]. A typical *component* here reduces to

$$\int_{1000}^{T}G(t)e^{iF(t)}dt$$

with $G(t)=t^{2\varepsilon-1/2}$ and $F(t)=-2t\ln t+2t+t\ln[\pi^2 x|S^{-1}[m_0]|]$. The result in [15] is applied to $[T2^{-k-1}, T2^{-k}]$ for $k\leqq\log_2 T$. *Each* interval of this type splits into $O(1)$ "admissible" subintervals. We conclude that:

$$\frac{1}{2\pi i}\int_{-\varepsilon-iT}^{-\varepsilon+iT}F_\mu(\xi\,;S)\frac{(2\pi x)^{\xi+1}}{\xi(\xi+1)}d\xi=O\left[\frac{x^{1-\varepsilon}}{\varepsilon}T^{2\varepsilon}\ln T\right].$$

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