91. *Uniqueness and Non-Uniqueness in the Cauchy Problem for a Class of Operators of Degenerate Type. II*

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In this note, we shall consider uniqueness and non-uniqueness of C^∞-solutions of the Cauchy problem for a class of partial differential operators whose characteristics degenerate infinitely on the initial surfaces. For weakly hyperbolic operators of this type, many authors studied on the well-posedness of the Cauchy problem. See [2], [3] and [4] for example. Our sufficient condition for uniqueness on the lower order terms corresponds to that of [2]. Theorem 2 shows that this condition is, in a sense, necessary for uniqueness. Note that we assume here only C^0-regularity on the coefficients of the lower order terms of operators (see (A.2)). Considering the sharp results of [3] and [4], we may conclude that uniqueness depends also on the regularity of the coefficients of operators.

§1. Preliminaries. In order to describe the degeneracy of characteristics on the initial surfaces, we prepare some results. The argument in this section is due to Tahara [2].

Let $\mu(t)$ be a function on $[0, T]$ satisfying

(1.1) $\mu(t)>0$ for $t>0$, $\mu(t)=O(t)$ as $t\longrightarrow +0$,

(1.2) $\mu(t)\in C^1([0, T])\cap C^\infty((0, T])$,

(1.3) $\mu(t)^k$, $\mu(t)^k\mu'(t)\in C^k([0, T])$ for any $k\in N$.

We define $\sigma(t)$ by $\sigma(t)=\exp\left(-\int_t^T \mu(s)^{-1}\,ds\right)$. Then we have

Lemma 1 (Tahara [2], Prop. 6.4). *The following conditions are equivalent to each other*:

(1.4) $\mu(t)=o(t)$ *as* $t\longrightarrow +0$,

(1.5) $\sigma(t)=O(t^m)$ *as* $t\longrightarrow +0$ *for any* $m\geqq 0$,

(1.6) $\mu(t)^m\sigma(t)\in C^\infty([0, T])$ *for any* $m\in \boldsymbol{Z}$.

Note that (1.5) implies that $t=0$ is a zero of infinite order of the function $\sigma(t)$. In what follows we assume that $\mu(t)$ and $\sigma(t)$ satisfy above conditions. And we continue $\mu(t)$ and $\sigma(t)$ smoothly to $t<0$.

Example. The functions

$$\mu(t)=t^{k+1}/k,\quad t/(-\log t)^k,\quad t^{k+1}\exp(-t^{-k})/k,\quad (k>0)$$

correspond respectively to

$$\sigma(t)=\exp(-t^{-k}),\quad \exp\{-(-\log t)^{k+1}/(k+1)\},\quad \exp\{-\exp(t^{-k})\}.$$