89. *Fourier-Mehler Transforms of Generalized Brownian Functionals**)

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1. **Generalized Brownian functionals.** In the continuous embeddings $\mathcal{S}\subset L^2(\boldsymbol{R})\subset\mathcal{S}^*$, $\mathcal{S}$ and $\mathcal{S}^*$ are the nuclear spaces of rapidly decreasing functions and tempered distributions, respectively. Let μ be the white noise measure on $\mathcal{S}^*$, i.e. its characteristic functional is given by

$$\int_{\mathcal{S}^*} \exp\,[i\langle \dot{B}, \xi\rangle]d\mu(\dot{B}) = \exp\,[-\|\xi\|^2/2] \equiv C(\xi), \qquad \xi\in\mathcal{S},$$

where $\|\cdot\|$ is the $L^2(\boldsymbol{R})$-norm. Being motivated by the well-known Wiener-Ito decomposition of $L^2(\mathcal{S}^*)$, Hida [1], [3] has introduced the following space $(L^2)^-$ of generalized Brownian functionals:

$$(L^2)^- = \sum_{n=0}^{\infty} \oplus K_n^{(-n)},$$

where $K_n^{(-n)}$ consists of generalized multiple Wiener integrals [2]. An element φ in $K_n^{(-n)}$ is realized as a distribution on $\boldsymbol{R}^n$ through the integral transform $\mathcal{T}$:

$$\begin{aligned}(\mathcal{T}\varphi)(\xi) &= \int_{\mathcal{S}^*} \exp\,[i\langle \dot{B}, \xi\rangle]\varphi(\dot{B})d\mu(\dot{B})\\ &= i^n C(\xi)\int_{\boldsymbol{R}^n} f(u_1,\cdots,u_n)\xi(u_1)\cdots\xi(u_n)du_1\cdots du_n, \qquad \xi\in\mathcal{S},\end{aligned}$$

where f is in the Sobolev space $\hat{H}^{-(n+1)/2}(\boldsymbol{R}^n)$.

2. **Renormalization.** Let T be a finite interval in $\boldsymbol{R}$. By using the renormalization procedure, we obtain the following three generalized Brownian functionals:

1) $\varphi(\dot{B}) = :\exp\left[\lambda\dot{B}(t) + c\int_T \dot{B}(u)^2du\right]:$, $t\in T$, $\lambda, c\in\boldsymbol{C}$, $c\neq 1/2$. The $\mathcal{T}$-transform of φ is given by

$$(\mathcal{T}\varphi)(\xi) = C(\xi)\exp\left[\frac{i\lambda}{1-2c}\xi(t) + \frac{c}{2c-1}\int_T \xi(u)^2du\right], \qquad \xi\in\mathcal{S}.$$

2) $\psi(\dot{B}) = :H_n\left(\dot{B}(t); \frac{1}{(1-2c)dt}\right)\exp\left[c\int_T \dot{B}(u)^2du\right]:$, $c\in\boldsymbol{C}$, $c\neq 1/2$.

The $\mathcal{T}$-transform of ψ is given by

$$(\mathcal{T}\psi)(\xi) = \frac{1}{n!}C(\xi)\left(\frac{i\xi(t)}{1-2c}\right)^n \exp\left[\frac{c}{2c-1}\int_T \xi(u)^2du\right], \qquad \xi\in\mathcal{S}.$$

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