87. *Degeneration of Surfaces with Trivial Canonical Bundles*

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The purpose of this note is to study a degeneration of surfaces with trivial canonical bundles, especially one which contains a surface of class VII in its singular fiber. Details will be published elsewhere. I would like to thank Prof. K. Ueno for his invaluable suggestions and encouragement.

§ 1. Let $\pi: X \to \varDelta$ be a proper surjective holomorphic map of a three dimensional complex manifold X to a disk $\varDelta = \{t \in C \mid |t| < \varepsilon\}$ with connected fibers. Assume that π is smooth at each point of $\pi^{-1}(\varDelta^*)$, $\varDelta^* = \varDelta - \{0\}$. We call such a holomorphic map $\pi: X \to \varDelta$ a *degeneration of surfaces* (or briefly, a *degeneration*). By a singular fiber X_0, we mean a divisor on X defined by $\pi = 0$. A smooth fiber $X_t = \pi^{-1}(t)$ $(t \neq 0)$ is called a general fiber.

A degeneration $\pi': X' \to \varDelta$ is called a modification of a degeneration $\pi: X \to \varDelta$, if there exists a bimeromorphic map $\varPhi: X \dashrightarrow X'$ such that the diagram

$$\begin{array}{ccc} X & \overset{\varPhi}{\dashrightarrow} & X' \\ & {}_{\pi}\searrow \quad \swarrow_{\pi'} & \\ & \varDelta & \end{array}$$

is commutative and over $\varDelta^*$, $\operatorname{res} \varPhi: \pi^{-1}(\varDelta^*) \to \pi'^{-1}(\varDelta^*)$ is biholomorphic.

A degeneration $\pi: X \to \varDelta$ is called semi-stable, if the singular fiber X_0 is a reduced divisor with simple normal crossings. Note that by Mumford's theorem every degeneration can be made semi-stable after a base change and a modification.

In this note, we shall study degenerations of surfaces up to modifications. We are mainly interested in a semi-stable degeneration of $K3$ surfaces which is not assumed to be projective nor Kähler.

§ 2. **Theorem.** *Let $\pi: X \to \varDelta$ be a semi-stable degeneration of $K3$ surfaces. Then this satisfies one of the following conditions:*

(i) *there exists a modification $\pi': X' \to \varDelta$ of $\pi: X \to \varDelta$ such that π' is also semi-stable and the canonical bundle $K_{X'}$ on X' is trivial.*

(ii) *one of the components of the singular fiber X_0 is a Hopf surface of its blown-up surface.*