# 86. *Free Arrangements of Hyperplanes over an Arbitrary Field*[*)]

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In [6], we proved a factorization theorem for the Poincaré polynomial of the complement of hyperplanes in an $l$-dimensional vector space over the complex number field $C$ when the arrangement of the hyperplanes is free. That was called Shephard-Todd-Brieskorn theorem there. Our main aim here is to report a generalized factorization theorem for a free arrangement over an arbitrary field. The detailed proof will appear in [3].

**1.** Let $A$ be an arrangement in an $l$-dimensional vector space $V$ over a field $K$. In other words, $A$ is a finite family of $(l-1)$-dimensional vector subspaces of $V$. Denote the dual vector space of $V$ by $V^*$. Let $S=S(V^*)$ be the symmetric algebra of $V^*$. Fix a base $\{x_1, \cdots, x_l\}$ for $V^*$, and $S$ is isomorphic to the polynomial algebra $K[x_1, \cdots, x_l]$. Let $Q \in S$ be a reduced defining equation for $\bigcup_{H \in A} H$. Then $Q$ is a product of elements of $V^*$. The derivation of $S$ is a $K$-linear map $\theta : S \to S$ satisfying $\theta|_K \equiv 0$ and $\theta(fg)=f\theta(g)+g\theta(f)$ for any $f, g \in S$.

**Definition 1.** *A derivation along A* (which is called a logarithmic vector field [4] when we are in the complex analytic category) is a derivation $\theta$ of $S$ satisfying

$$\theta(Q) \in QS.$$

Let $D(A)$ denote the set of derivations along $A$. Then $D(A)$ is naturally an $S$-module.

**Definition 2.** If $D(A)$ is an $S$-free module, we say that $A$ is *a free arrangement.*

**Definition 3.** A derivation $\theta$ of $S$ is said to be *homogeneous of degree b* if $\theta(x_i) \in S_b$ $(i=1, \cdots, l)$, where $S_b$ is the vector subspace of $S$ generated by monomials of degree $b$. We write $b=\deg \theta$. We can show that $D(A)$ has a free base $\{\theta_1, \cdots, \theta_l\}$ consisting of homogeneous derivations if $A$ is a free arrangement. The integers $(\deg \theta_1, \cdots, \deg \theta_l)$ are called *the degree* of $A$ (called the generalized exponents of $A$ in [6]). They depend only upon $A$.

The following useful criterion, proved by K. Saito [4] when $K=C$, remains true for arbitrary $K$:

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