84. S^1 Actions with Only Isolated Fixed Points on Almost Complex Manifolds

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1. The purpose of this note is to announce some results on S^1 actions having only isolated fixed points on an almost complex manifold admitting a quasi-ample complex line bundle. Details will appear elsewhere.

Let M be an oriented, connected, closed C^∞ manifold on which a smooth action of S^1 is given such that its fixed points are all isolated. Then the fixed point set consists of exactly χ points $\{P_i\}$ where χ denotes the Euler number of M. Let E be a complex line bundle over M such that the given S^1 action can be lifted to an action on the line bundle E. We call such a line bundle admissible. If E is an admissible line bundle, then we fix a lifting of the action on E and consider the fiber E_{P_i} of E over a fixed point P_i. E_{P_i} is a complex S^1-module so that it can be written in the form

$$E_{P_i}=t^{a_i} \tag{1.1}$$

where t denotes the standard 1 dimensional S^1-module. The integer a_i will be called the weight of E at P_i. We note that if we choose another lifting of action then the weights a_i are changed simultaneously to a_i+a for some a. An admissible line bundle over an even dimensional manifold M will be called quasi-ample if the weights a_i are all different and

$$(c_1(E))^n[M]\neq 0, \qquad \dim M=2n,$$

where $c_1(E)$ denotes the first Chern class of E.

Now we assume that M is an almost complex manifold and the action of S^1 preserves the almost complex structure. Such a manifold will be called almost complex S^1-manifold. Then, restricting the complex tangent bundle TM to each fixed point P_i, we get an S^1-module

$$TM\,|\,P_i=\sum_k t^{m_{ik}}$$

where the m_{ik} are non-zero integers. These integers m_{ik} are called weights of M at P_i. Later we shall consider the following condition (D) relating a quasi-ample line bundle E and the tangent bundle:

(D) There exist integers $k_0\geq 0$ and d such that the identity

$$\sum_k m_{ik}=k_0a_i+d$$

holds for all i.