77. *Three Commodity Flows in Graphs*

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Let $G=(V, E)$ be a graph (finite undirected, possibly with multiple edges but without loops). In this paper a path has no repeated edges, and we permit the path with one vertex and no edges. For two distinct vertices x, y we let $\lambda(x, y)=\lambda_G(x, y)$ be the maximal number of edge-disjoint paths between x and y, and we let $\lambda(x, x)=\infty$.

We first consider the following problem.

Let $(s_1, t_1), \cdots, (s_k, t_k)$ be pairs (not necessarily distinct) of vertices of G. When is the following true?

(1.1) There exist edge-disjoint paths $P_1, \cdots, P_k$ such that P_i has ends s_i, t_i $(1\leq i\leq k)$.

Seymour [9] and Thomassen [11] answered to this problem when $k=2$, and Seymour [9] when $s_1, \cdots, s_k, t_1, \cdots, t_k$ take only three distinct values.

Our result is the following

Theorem 1. *Suppose that $s_1, s_2, s_3, t_1, t_2, t_3$ are vertices of a graph G. If for each $i=1, 2, 3$ $\lambda(s_i, t_i)\geq 3$, then there exist edge-disjoint paths P_1, P_2, P_3 of G, such that P_i has ends s_i and t_i $(i=1, 2, 3)$.*

If $\lambda(s_i, t_i)\leq 2$ for some i, then this conclusion does not hold. Fig. 1 gives a counterexample.

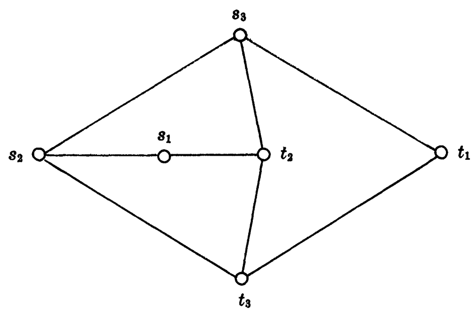

Fig. 1

For a positive integer k, we let $g(k)$ be the smallest integer such that for every $g(k)$-edge-connected graph and for every vertices $s_1, \cdots, s_k, t_1, \cdots, t_k$ of the graph, (1.1) holds. Thomassen [11] conjectured the following.

Conjecture (Thomassen). *For each odd integer $k\geq 1$, $g(k)=k$, and for each even integer $k\geq 2$, $g(k)=k+1$.*