72. G-Vector Bundles and F-Projective Modules*)

By Katsuo KAWAKUBO
Osaka University

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§1. **Introduction.** Swan has shown that there is a one to one correspondence between vector bundles over a compact Hausdorff space X and finitely generated projective modules over the ring of continuous real-valued functions on X [7].

In the present paper, we will consider an equivariant version of this. Let G be a compact topological group. Then a notion of G-vector bundles is already defined [1]. On the other hand, we introduce notions of equivariant modules, of a family F of equivariant modules and of F-projective modules so that we have an equivariant Swan theorem.

For each family F, we define two kinds of equivariant algebraic K-theories associated with F. Taking a suitable family F, we have an isomorphism of an equivariant topological K-theory and our equivariant algebraic K-theory associated with F.

Equivariant algebraic K-theory is studied along the line of Quillen [6] by Fiedorowicz, Hauschild and May [4], while our approach is along the line of the classical algebraic K-theory [5]. The reason will clear up in a subsequent paper. Namely we will show that our equivariant algebraic K-theory is a Mackey functor [3]. Accordingly the Dress induction theorem [2] is applicable. Using our equivariant Swan theorem, we will show that Brauer and Artin type induction theorems hold in equivariant topological K-theories $KO_G(X)$ and $K_G(X)$. Accordingly equivariant topological K-theories are characterized by the hyperelementary subgroups.

§2. **Families and equivariant algebraic K-theory.** The word *ring* will always mean associative ring with an identity element 1. Let G be a group. A *G-ring* is a ring Λ together with a G-action on Λ preserving the ring structure. If Λ is a G-ring, a *ΛG-module* is a module M over Λ together with a G-action on M such that

$$(*) \qquad g(\lambda_1 m_1+\lambda_2 m_2)=(g\lambda_1)(gm_1)+(g\lambda_2)(gm_2)$$
$$\text{for any } g\in G,\ \lambda_i\in\Lambda,\ m_i\in M.$$

A collection F of finitely generated ΛG-modules is called a *family* if the following holds;

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