69. On the Asymptotic Behavior of a Nonlinear Contraction Semigroup and the Resolvent Iteration

By Tetsuji SUGIMOTO and Munehito KOIZUMI
Department of Mathematics, Waseda University

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1. Introduction. Throughout this note X denotes a real Banach space, A is an m-dissipative operator in X and $\{T(t): t \geqq 0\}$ is the contraction semigroup on $\overline{D(A)}$ (the closure of the domain of A) generated by A. For $r>0$, J_r denotes the resolvent of A, i.e., $J_r=(I-rA)^{-1}$.

Consider the resolvent iteration

$$\text{(RI)} \qquad \begin{cases} x_0 \in X \\ x_n = J_{r_n} x_{n-1} \qquad \text{for } n \geqq 1 \end{cases}$$

where $\{r_n\}$ is a sequence of positive numbers. The purpose of this note is to prove the following

Theorem. *$T(t)x$ is strongly* (resp. weakly) *convergent as $t \to \infty$ for all $x \in \overline{D(A)}$ if and only if* (RI) *is strongly* (resp. weakly) *convergent as $n \to \infty$ for all $x_0 \in X$ and all $\{r_n\} \in l^2 \backslash l^1$.*

This theorem has been proved by Passty [1, Theorem 2] under an additional assumption that A is Lipschitzian. We can, however, remove the assumption on A by using the idea of [3].

2. Proof of Theorem. By a *contractive evolution system* on $C(\subset X)$ we mean a two-parameter family $\{U(t, s): 0 \leqq s \leqq t < \infty\}$ of self-maps of C satisfying: (i) $U(t, t)z = z$ for $t \in R^+ = [0, \infty)$ and $z \in C$; (ii) $U(t, s)\ U(s, r)z = U(t, r)z$ for $t \geqq s \geqq r$ in R^+ and $z \in C$; (iii) $\|U(t, s)z_1 - U(t, s)z_2\| \leqq \|z_1 - z_2\|$ for $t \geqq s$ in R^+ and $z_1, z_2 \in C$.

Definition ([1]). A contractive evolution system $\{U(t, s): 0 \leqq s \leqq t < \infty\}$ on $\overline{D(A)}$ is said to be asymptotically equal to the semigroup $\{T(t): t \geqq 0\}$ if for each $x \in \overline{D(A)}$,

(2.1) $\lim_{t\to\infty} \|U(t+h, s)x - T(h)U(t, s)x\| = 0$ for each $s \geqq 0$, uniformly in $h \geqq 0$ and

(2.2) $\lim_{t\to\infty} \|U(t+h, t)T(t)x - T(t+h)x\| = 0$ uniformly in $h \geqq 0$.

The following proposition is due to Passty [1].

Proposition 2.1. *Let $\{U(t, s): 0 \leqq s \leqq t < \infty\}$ be a contractive evolution system which is asymptotically equal to the semigroup $\{T(t): t \geqq 0\}$. Then $T(t)x$ is strongly* (resp. weakly) *convergent as $t \to \infty$ for all $x \in \overline{D(A)}$ if and only if $U(t, s)x$ is strongly* (resp. weakly) *convergent as $t \to \infty$ for all $x \in \overline{D(A)}$ and all $s \geqq 0$.*