65. *Wave Front Solution of Some Competition Models with Migration Effect*

By Yuzo HOSONO*) and Syozo NIIZEKI**)

(Communicated by Kôsaku YOSIDA, M. J. A., June 14, 1983)

1. Introduction. Consider the following semilinear hyperbolic system of partial differential equations

$$(1)\quad \begin{cases} \dfrac{\partial u}{\partial t}+\mu\dfrac{\partial u}{\partial x}=\left(a_1-b_1u-\dfrac{c_1v}{1+e_1u}\right)u\equiv f(u,v)u \\ \dfrac{\partial v}{\partial t}+\nu\dfrac{\partial v}{\partial x}=\left(a_2-\dfrac{b_2u}{1+e_2v}-c_2v\right)v\equiv g(u,v)v, \\ \qquad\qquad (x,t)\in(-\infty,+\infty)\times(0,+\infty), \end{cases}$$

where all the coefficients in (1) are real constants such that $\mu<\nu$, $a_i>0$, $b_i>0$, $c_i>0$ and $e_i\geqq 0$ $(i=1,2)$. Here, u and v are population densities of two species with distinct migration speed μ and ν respectively. Therefore, we consider nonnegative solutions only. In case of $e_1=e_2=0$, the system (1) becomes the classic Volterra-Lotka competition models. Yamaguti [4] considered the system (1) when f and g are linear functions of u and v, and he obtained the exact solutions by using Hirota's method [2]. By the computer simulations, we have found that the solutions given in [4] include the wave front solutions of the form

$$u(x,t)=u(z)=\left\{\gamma\frac{a_1}{b_1}\exp z\right\}\Big/\{1+\gamma\exp z\},$$

$$v(x,t)=v(z)=\frac{a_2}{c_2}\Big/\{1+\gamma\exp z\},\qquad z=Qx-\omega t,$$

where γ, Q and ω are some constants.

In this paper we shall show that the system (1) has unique (except modulo translation) wave front solutions joining two distinct states $P_1=(a_1/b_1,0)$ and $P_2=(0,a_2/c_2)$, where only one of two species exists.

2. Formulation of problem. In order to seek wave front solutions joining P_2 and P_1, put $(u(x,t),v(x,t))=(u(z),v(z))$ with $z=x-\sigma t$ $(\sigma\neq\mu,\nu)$. Then our problem is reduced to find solutions of ordinary differential equations of the form

$$(2)\quad \frac{du}{dz}=\frac{1}{\mu-\sigma}f(u,v)u\equiv f_1(u,v),\qquad \frac{dv}{dz}=\frac{1}{\nu-\sigma}g(u,v)v\equiv g_1(u,v)$$

with the boundary conditions

*) Department of Computer Science, Kyoto Sangyo University.
**) Department of Mathematics, Faculty of Science, Kochi University.