

64. On the Isomonodromic Deformation of a Linear Ordinary Differential Equation of the Third Order

By Hironobu KIMURA

Department of Mathematics, University of Tokyo

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§ 1. Introduction. Consider a third order linear ordinary differential equation of Fuchsian type

$$(1.1) \quad \frac{d^3y}{dx^3} + p_1 \frac{d^2y}{dx^2} + p_2 \frac{dy}{dx} + p_3 y = 0$$

with the following Riemannian scheme :

$$(1.2) \quad \left\{ \begin{array}{ccccc} x=0 & x=1 & x=t & x=\lambda_j & x=\infty \\ \alpha_0 & \alpha_1 & \beta & \gamma_j & \alpha_\infty \\ \alpha_0 + \kappa_0 & \alpha_1 + \kappa_1 & \beta + \theta & \gamma_j + 2 & \alpha_\infty + \kappa_\infty \\ \alpha_0 + \kappa'_0 & \alpha_1 + \kappa'_1 & \beta + \theta' & \gamma_j + 3 & \alpha_\infty + \kappa'_\infty \end{array} \right\} \\ (j=1, 2, 3, 4)$$

and we suppose that the singularities $x=\lambda_j$ ($j=1, 2, 3, 4$) are non-logarithmic ones and the characteristic exponents at each singular point do not differ by integer.

The purpose of this paper is to derive a system of isomonodromic deformation equations of (1.1) regarding t as deformation parameter.

§ 2. Hamiltonian system attached to (1.1). The coefficients $p_j(x)$ ($j=1, 2, 3$) of the equation (1.1) are given by

$$\begin{aligned} p_1(x) &= \frac{a_0^1}{x} + \frac{a_1^1}{x-1} + \frac{b^1}{x-t} + \sum_{k=1}^4 \frac{c_k^1}{x-\lambda_k}, \\ p_2(x) &= \frac{a_0^2}{x^2} + \frac{a_1^2}{(x-1)^2} + \frac{b^2}{(x-t)^2} + \sum_{k=1}^4 \frac{c_k^2}{(x-\lambda_k)^2} \\ &\quad + \frac{a_\infty^2}{x(x-1)} - \frac{t(t-1)H}{x(x-1)(x-t)} + \sum_{k=1}^4 \frac{\lambda_k(\lambda_k-1)\mu_k}{x(x-1)(x-\lambda_k)}, \\ p_3(x) &= \frac{a_0^3}{x^3} + \frac{a_1^3}{(x-1)^3} + \frac{b^3}{(x-t)^3} + \sum_{k=1}^4 \frac{c_k^3}{(x-\lambda_k)^3} \\ &\quad + \frac{1}{T(x)} \left[a_\infty^3 + \eta_0 \frac{t}{x} - \eta_1 \frac{t-1}{x-1} + \eta_t \frac{t(t-1)}{x-t} \right. \\ &\quad \left. + \sum_{k=1}^4 \left\{ \frac{T(\lambda_k)}{(x-\lambda_k)^2} + \frac{T'(\lambda_k)}{x-\lambda_k} \right\} \xi_k + \sum_{k=1}^4 \frac{\zeta_k}{x-\lambda_k} \right], \end{aligned}$$

where

$$T(x) = x(x-1)(x-t)$$

and a_i^j, b^j, c_k^j ($i=1, 2, 3; k=1, 2, 3, 4; j=0, 1, \infty$) are constants de-