62. On the Resolution Process of Normal Gorenstein Surface Singularity with $p_a \leqq 1$

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Let (V, p) be a normal Gorenstein surface singularity over $\boldsymbol{C}$ and ψ ; $(\tilde{V}, A) \to (V, p)$ be a resolution of (V, p) with the exceptional set A. The arithemetic genus of (V, p) is the integer $p_a(V, p)$ defined by $p_a(V, p) = \sup\{p_a(D) \mid \text{divisor } D \text{ on } \tilde{V},\ D>0,\ |D| \subset A\}$. The geometric genus of (V, p) is the integer $p_g(V, p)$ defined by $p_g(V, p) = \dim_C R^1\psi_* O_{\tilde{V}}$ (see [6]).

The singularity (V, p) is called *rational* (resp. *elliptic*) if $p_a(V, p) = 0$ (resp. $p_a(V, p) = 1$). The *Zariski's canonical resolution*, which we simply call Z.C.R. here, of (V, p) is the resolution obtained by the composition of blowing-up with center maximal ideal followed by normalization (see e.g., [3]). The singularity (V, p) is called *absolutely isolated* if all the normalizations in Z.C.R. of (V, p) are trivial. The purpose of this note is to announce the recent results on the Z.C.R. for the normal Gorenstein surface singularity with $p_a \leqq 1$. The details will be published elsewhere.

§1. Absolute isolatedness of elliptic singularity. Theorem 1. *Let (V, p) be a normal Gorenstein surface singularity with $p_a \leqq 1$. Then Z.C.R. is obtained by the composition of blowing-ups as follows:*

$$(*)\qquad \begin{array}{ccccccccc} U=U_0 & \longleftarrow & U_1 & \longleftarrow & U_2 & \longleftarrow & \cdots & \longleftarrow & U_N \\ \cup & & \cup & & \cup & & & & \cup \\ & \psi_1 & & \psi_2 & & \psi_3 & & \psi_N & \\ V=V_0 & \longleftarrow & V_1 & \longleftarrow & V_2 & \longleftarrow & \cdots & \longleftarrow & V_N \end{array}$$

where $V \subset U$ is the minimal embedding, ψ_i the blowing-up of U_{i-1} with smooth center $C_i \subset V_{i-1}$, and V_i the strict transformation of V_{i-1}, $1 \leqq i \leqq N$. Moreover we have: There is an integer M $(\leqq N)$ such that (i) *V_k is normal for $k \leqq M$.* (ii) *ψ_k is a blowing-up with point center p_k, such that (V_{k-1}, p_k) is Gorenstein of maximal embedding dimension* [4] *of multiplicity $\geqq 3$ for $k \leqq M$.* (iii) *At each stage, in which V_i is normal, there is at most one non-rational singularity.* (iv) $\mathrm{mult}_q V_M \leqq 2$ *for any point $q \in V_M$.* (v) [5] *In the Z.C.R. for the singularity of V_M, the normalizations are obtained by one blowing-up along* (reduced) $\boldsymbol{P}^1$.

There are $p_g(V, p) - M$ blowing-ups along $\boldsymbol{P}^1$ in the diagram $(*)$. On the other hand, we have

Theorem 2 [1]**.** *Let (V, p) be a normal surface singularity of multiplicity two. Then (V, p) is absolutely isolated if and only if (V, p)*