7. On a Question Posed by Huckaba-Papick

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1. Introduction. Let R be a commutative integral domain with identity, and let x be an indeterminate. By $c(f)$ we denote the ideal of R generated by the coefficients of f for an element f of $R[x]$ (the 'content' of f). Let K be the quotient field of R. We denote the R-submodule $\{b \in K; b\mathfrak{A} \subset R\}$ of K for an ideal $\mathfrak{A}$ of R by $\mathfrak{A}^{-1}$. We set $S=\{f \in R[x]; c(f)=R\}$ and $U=\{f \in R[x]; c(f)^{-1}=R\}$. These are multiplicative systems of $R[x]$. Hence we can define subrings $R[x]_S$ and $R[x]_U$ of $K(x)$; $R[x]_S \subset R[x]_U$. If we have $\mathfrak{A}\mathfrak{A}^{-1}=R$ for each finitely generated ideal $\mathfrak{A}$ of R, R is said to be a *Prüfer ring*. If each finitely generated ideal of R is a principal ideal, R is said to be a *Bezout ring*. If R is a Bezout ring, then it is a Prüfer ring. Huckaba-Papick studied in [2], the following problems: When does $R[x]_S=R[x]_U$ hold? and when is $R[x]_U$ a Prüfer ring? And they posed the open question:

Question ([2, Remark (3.4), (c)]). If $R[x]_U$ is a Prüfer ring, is it a Bezout ring?

The main purpose of this paper is to give an affirmative answer to this question. We prove the following result:

Theorem 1. *If $R[x]_U$ is a Prüfer ring, it is a Bezout ring.*

Among other things, Huckaba-Papick prove the following result in [2, Theorem (3.1), (c)]: If R is a Krull ring, then $R[x]_U$ is a Bezout ring. But their proof does not seem to be complete. So we prove the following result for the sake of completeness.

Theorem 2. *If R is a Krull ring, then $R[x]_U$ is a principal ideal ring. Conversely, if $R[x]_U$ is a Krull ring, then R is also a Krull ring.*

2. Proofs of Theorems 1 and 2. We denote the ideal $\{r \in R; rb \in (a)\}$ of R for two elements a, b of R by $(a:b)$ as in [2]. Let $\mathcal{P}(R)$ be the set of prime ideals of R which are minimal prime ideals over $(a:b)$ for some elements a, b of R.

Lemma 3 ([3, Theorem E]).

(1) $U=R[x]-\bigcup_{P \in \mathcal{P}(R)} PR[x]$;

(2) $R=\bigcap_{P \in \mathcal{P}(R)} R_P$.

Lemma 4 ([1, § 18, Exercise 12]). *Let V be a valuation ring of $K(x)$ of the form $R[x]_Q$ for a prime ideal Q of $R[x]$. Then we have either* (1) *or* (2) *of the following*: