60. Lie Groups and Lie Algebras with generalized Bose-Fermi Symmetric Parameters

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1. Introduction. Lie superalgebras and Lie supergroups have been utilized extensively in physics. In those theories, Grassmann algebras play an important role. In fact, the theory of supermanifolds and supergroups are formulated on the basis of Grassmann algebras (Rogers [5], [6]). They have been also used in describing Fermi systems (Berezin [1]). Ohnuki and Kamefuchi [3] have considered generalized Grassmann numbers and generalized Bose numbers to describe para-Fermi and para-Bose systems (see also [4]). Scheunert [7] introduced algebras satisfying certain conditions which are considered to be the most general with respect to the concept of commutativity. We call them the σ-commutative algebras.

In this paper, we develop the theory of matrices whose entries are elements of a σ-commutative algebra and study the groups and the algebras consisting of those matrices. The details of the results will appear elsewhere.

2. σ-symmetry. In this section, we give some basic concepts which were basically formulated by Scheunert [7].

Let k be a field and let G be an abelian group. A mapping $\sigma: G \times G \to k$ is called a *sign* (commutation factor in terms of [7]) of G, if it satisfies

(1) $\sigma(\alpha+\beta, \gamma)=\sigma(\alpha, \gamma)\sigma(\beta, \gamma)$,

(2) $\sigma(\alpha, \beta)\sigma(\beta, \alpha)=1$,

for any $\alpha, \beta, \gamma \in G$. The pair (G, σ) is called a *signed group.*

Throughout this paper, a field k and a signed group (G, σ) are fixed. As is easily seen, $\sigma(\alpha, \alpha)$ is either 1 or -1 for any $\alpha \in G$. An element α of G such that $\sigma(\alpha, \alpha)=1$ (resp. -1) is called *even* (resp. *odd*).

A G-graded (associative) algebra $A=\oplus_{\alpha\in G} A_\alpha$ is called *σ-commutative* or *σ-symmetric* if $ab=\sigma(\alpha, \beta)ba$ for any $\alpha, \beta \in G$, $a \in A_\alpha$, $b \in A_\beta$.

Let $V=\oplus_{\alpha\in G} V$ be a G-graded vector space over k. Let $T(V)$ be the tensor algebra of V over k and I be the ideal of $T(V)$ generated by the elements of the form $x\otimes y-\sigma(\alpha, \beta)y\otimes x$ with $x \in V_\alpha$ and $y \in V_\beta$. The

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