57. *Riemann-Hilbert-Birkhoff Problem for Integrable Connections with Irregular Singular Points*

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Let M be a complex manifold and let H be a divisor on M. Denote by $\Omega^p(*H)$ the sheaf over M of germs of meromorphic p-forms which are holomorphic in $M-H$ and have poles on H for $p=0, \cdots, n$. In case $p=0$, we use frequently $\mathcal{O}(*H)$ instead of $\Omega^0(*H)$.

We suppose throughout this paper that the divisor H has at most normal crossings.

Let $\mathcal{S}$ be a locally free sheaf of $\mathcal{O}(*H)$-modules of rank m and let ∇ be an integrable connection on $\mathcal{S}$. For any point $p \in H$, there exists an open set U in M containing p and a free basis $e_U=(e_{1U}, \cdots, e_{mU})$ of $\mathcal{S}$ over U. With respect to the free basis e_U, the connection ∇ is represented by $(d+\Omega_{eU})$, i.e.

$$\nabla(\langle e_{1U}, \cdots, e_{mU}\rangle u)=\langle e_{1U}, \cdots, e_{mU}\rangle(du+\Omega_{eU}u),$$

where Ω_{eU} is an m-by-m matrix of meromorphic 1-forms with poles at most on H and u is any m-vector of functions in $\mathcal{O}(*H)(U)$. If $f_U=\langle f_{1U}, \cdots, f_{mU}\rangle$ is another free basis of $\mathcal{S}$ over U, then there exists an m-by-m invertible matrix G of functions in $\mathcal{O}(*H)(U)$ such that

$$\langle f_{1U}, \cdots, f_{mU}\rangle=\langle e_{1U}, \cdots, e_{mU}\rangle G,$$

$$\nabla(\langle f_{1U}, \cdots, f_{mU}\rangle u)=\langle f_{1U}, \cdots, f_{mU}\rangle(du+(G^{-1}\{\Omega_{eU}G+dG\})u).$$

Let $x_1, \cdots, x_m$ be holomorphic local coordinates at p on U with $U\cap H=\{x_1\cdots x_{n''}=0\}$, then Ω_{eU} is written of the form

$$\Omega_{eU}=\sum_{i=1}^{n''} x^{-p_i}x_i^{-1}A_i(x)dx_i+\sum_{i=n''+1}^{n} x^{-p_i}A_i(x)dx_i,$$

where $p_i=(p_{i1}, \cdots, p_{in''}, 0, \cdots, 0) \in N^n$ and $A_i(x)$ is an m-by-m matrix of holomorphic functions in U for $i=1, \cdots, n$, and Ω_{eU} satisfies, by the integrability condition, $d\Omega_{eU}+\Omega_{eU}\wedge\Omega_{eU}=0$.

Suppose that for any point p on H

(H♯) *there exists an open set U containing p with holomorphic coordinates $x_1, \cdots, x_n$ and a free basis $\langle e_{1U}, \cdots, e_{mU}\rangle$ of $\mathcal{S}$ such that Ω_{eU} is written of the above form satisfying*

(H♯1) *$p_i=0$ or, $p_i>0$ and $A_i(0)$ has m distinct eigenvalues for all $i=1, \cdots, n''$.*

Let M^- be the real blow-up along H of M with the natural projection $pr: M^-\to M$. Denote by $\mathcal{A}^-$ the sheaf over M^- of germs of functions strongly asymptotically developable and write $\mathcal{A}^-(*H)$ for $\mathcal{A}^-\otimes_{pr^*\mathcal{O}} pr^*\mathcal{O}(*H)$. Denote by $GL(m, \mathcal{A}^-)$ and $GL(m, \mathcal{A}^-(*H))$ the