55. The Exponential Calculus of Microdifferential Operators of Infinite Order. IV

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1. Introduction. In this note we show that there is a formal symbol q of order at most $1-0$ (see [1], [2] for the notation) satisfying

(1.1) $$\exp :p:=:\exp q:.$$

Here p is a formal symbol of order at most $1-0$. That is, the exponential of an operator has an exponential symbol. Such q can be calculated from p.

2. Exponential of operators. Let X be an open set in C^n with coordinates $x=(x_1,\cdots,x_n)$, Ω a conic open set in $T^*X\simeq X\times C^n_\xi$. Let $p(t;x,\xi)$ be a formal symbol of order at most $1-0$ defined in Ω. We shall consider the operator $\exp(s:p(t;x,\xi):)$ $(s\in C)$. Let us define a sequence $\{p^{(k)}(t;x,\xi)\}$ $(k=0,1,2,\cdots)$ of formal symbols by

(2.1) $$p^{(0)}(t;x,\xi)=1,$$

(2.2) $$p^{(k+1)}(t;x,\xi)=\exp(t\partial_\xi\cdot\partial_y)p(t;x,\xi)p^{(k)}(t;y,\eta)|_{\eta=\xi}^{y=x}.$$

Here $k=0,1,2,\cdots$. Then we set

(2.3) $$P(t;s,x,\xi)=\sum_{k=0}^{\infty}\frac{s^k}{k!}p^{(k)}(t;x,\xi).$$

Here $s\in C$. By the definition we have $:p^{(k)}(t;x,\xi):=(:p(t;x,\xi):)^k$. Therefore $P(t;s,x,\xi)$ formally satisfies the following differential equation:

(2.4) $$\partial_s P(t;s,x,\xi)=\exp(t\partial_\xi\cdot\partial_y)p(t;x,\xi)P(t;s,y,\eta)|_{\eta=\xi}^{y=x},$$

(2.5) $$P(t;0,x,\xi)=1.$$

Moreover we have

Proposition 1. *For every $s\in C$ the formal power series $P(t;s,x,\xi)$ in t is a formal symbol defined in Ω.*

Hence $P(t;s,x,\xi)$ defines an operator $:P(t;s,x,\xi):$ which satisfies

(2.6) $$\partial_s :P(t;s,x,\xi):=:p(t;x,\xi)::P(t;s,x,\xi):,$$

(2.7) $$:P(t;0,x,\xi):=1.$$

Therefore $\exp(s:p(t;x,\xi):)$ makes sense, which is defined by

(2.8) $$\exp(s:p(t;x,\xi):)=:P(t;s,x,\xi):.$$

3. Statement of the results. Let Ω be a conic open set in T^*X, $p(t;x,\xi)=\sum_{j=0}^{\infty}t^jp_j(x,\xi)$ a formal symbol of order at most $1-0$ defined in Ω. Let us define two sequences of symbols $\{\psi_{l,k}^{(j)}(x,y,\xi,\eta)\}$ and $\{q_k^{(j)}(x,\xi)\}$ defined respectively in $\Omega\times\Omega$ and in Ω by the following recursion formulae: