54. *Singularities of Solutions of the Hyperbolic Cauchy Problem in Gevrey Classes*

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1. Introduction. Singularities of solutions of the hyperbolic Cauchy problem have been investigated by many authors. In these works, Hamilton flows (null bicharacteristic flows) played a key role. However, in general, Hamilton flows can not be defined unless the characteristic roots are smooth. In [8], we generalized Hamilton flows. In this note, we shall give outer estimates of the wave front sets in Gevrey classes of solutions of the Cauchy problems for hyperbolic operators, whose principal parts have real analytic coefficients, using the generalized Hamilton flows.

2. Assumptions and results. Let $P(x, \xi)$ be a polynomial of $\xi=(\xi_1, \xi')=(\xi_1, \cdots, \xi_n)$ and write $P(x, \xi)=\sum_{j=0}^{m} P_j(x, \xi)$, where $P_j(x, \xi)$ is a homogeneous polynomial of degree j in ξ. Let $1<\kappa<\infty$ and denote by $\mathcal{E}^{(\kappa)}, \mathcal{E}^{\{\kappa\}}, \mathcal{D}^{(\kappa)}$ and $\mathcal{D}^{\{\kappa\}}$ spaces of ultradifferentiable functions on $\boldsymbol{R}^n$ (Gevrey classes) (see, e.g., Komatsu [5]). Moreover we denote by $\mathcal{D}^{(\kappa)\prime}$ and $\mathcal{D}^{\{\kappa\}\prime}$ spaces of ultradistributions (see [5]). Now let us state our assumptions:

(A-1) The coefficients of $P_m(x, \xi)$ are real analytic, defined for $x=(x_1, \cdots, x_n) \in \boldsymbol{R}^n$ and the coefficients of $P_j(x, \xi)$ $(j=0, \cdots, m-1)$ belong to $\mathcal{E}^{\{\kappa_1\}}$, where $1<\kappa_1<\infty$.

(A-2) $P_m(x, \xi)$ is hyperbolic with respect to $\vartheta=(1, 0, \cdots, 0) \in \boldsymbol{R}^n$, i.e.,

$$P_m(x, \xi-i\tau\vartheta)\neq 0 \qquad \text{for } x \in \boldsymbol{R}^n,\ \xi \in \boldsymbol{R}^n \text{ and } \tau>0.$$

(A-3) $1<\kappa_1<\kappa_0\equiv r/(r-1)$, where $r\geqq 2$ and the multiplicities of the roots of $P_m(x, \xi_1, \xi')=0$ in ξ_1 are not more than r when $x \in \boldsymbol{R}^n$ and $\xi' \in \boldsymbol{R}^{n-1}\setminus\{0\}$.

We shall consider the Cauchy problem

$$\text{(CP)} \qquad \begin{cases} P(x, D)u(x)=f(x), & x \in \boldsymbol{R}^n, \\ \operatorname{supp} u \subset \{x_1 \geqq 0\}, \end{cases}$$

where $f \in \mathcal{D}^{\{\kappa_1\}\prime}$ with $\operatorname{supp} f \subset \{x_1 \geqq 0\}$ and $D=-i(\partial/\partial x_1, \cdots, \partial/\partial x_n)$. First let us define the localization $P_{mz}(\delta z)$ at $z \in T^*\boldsymbol{R}^n$ by

$$P_m(z+s\delta z)=s^{\mu}(P_{mz}(\delta z)+o(1)) \qquad \text{as } s\to 0,$$

where $P_{mz}(\delta z)\not\equiv 0$ (in δz) is a (homogeneous) polynomial of $\delta z \in T_z(T^*\boldsymbol{R}^n)$. Then $P_{mz}(\delta z)$ is hyperbolic with respect to $(0, \vartheta) \in \boldsymbol{R}^{2n}$ (see [8]). Therefore we can define $\Gamma(P_{mz}, (0, \vartheta))$ as the connected component of the set