52. Group Factors of the Haagerup Type

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1. Let N be a type II_1 factor with the canonical trace τ. We call it a factor of the *Haagerup type* if there exists a net $(P_\alpha)_\alpha$ of normal linear maps on N which satisfy the following conditions;

(1) each P_α is completely positive on N,

(2) each P_α is compact (i.e. for any $\varepsilon>0$, there exists a finite dimensional linear map Q on N such that $\|P_\alpha(x)-Q(x)\|_2<\varepsilon\|x\|_2$ for all $x\in N$),

and

(3) $\|P_\alpha(x)-x\|_2\to 0$, for all $x\in N$.

Here, we put $\|x\|_2=\tau(x^*x)^{1/2}$ for $x\in N$.

This is a factor in the "Haagerup case" following A. Connes, and he remarked that each subfactor of a factor of the Haagerup type is again of the Haagerup type ([4]). Hence in the set of full II_1 factors, the class of the Haagerup type constitutes a minimal class.

In this paper, we shall characterize a property of an ICC group G, that its group von Neumann algebra $R(G)$ is to be of the Haagerup type. We shall call this property of the group the *property* (H). In [1], Akemann and Walter have investigated relations among various properties of locally compact groups, and they showed, in particular, that a group G does not have the property (T) of Kazhdan if G has the property (H). Now an application of our characterization shows that a group G may not have the property (H), even if G does not have the property (T). Another conclusion is that the full II_1 factors $R(F_2)$, $R(SL(3,Z))$ and $R(F_2\times SL(3,Z))$ are not isomorphic.

2. Let G be a discrete countable group. We denote by λ the left regular representation of $G:(\lambda(g)\xi)(h)=\xi(g^{-1}h)$ $(g,h\in G,\ \xi\in l^2(G))$. The group von Neumann algebra $R(G)$ is the von Neumann algebra on the Hilbert space $l^2(G)$ which is generated by $\{\lambda(g);g\in G\}$. The algebra $R(G)$ is a type II_1 factor if and only if G is an ICC group (i.e. the class $\{hgh^{-1};h\in G\}$ is infinite for each $g\in G\backslash\{1\}$, where 1 is the identity of G). For a $g\in G$, let $\delta(g)$ be the characteristic function of $\{g\}$. Then the factor $R(G)$ has the unique trace τ defined by $\tau(x)=(x\delta(1),\delta(1))$ for all $x\in R(G)$. Each $x\in R(G)$ has a unique form $x=\sum_{g\in G}x(g)\lambda(g)$ ($x(g)$ is a scalar for all $g\in G$) in the sense of $\|\cdot\|_2$-metric convergence.

Definition. A countable infinite group G is said to have the