6. Global Solutions of the Nonlinear Schrödinger Equation in Exterior Domains

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§ 1. **Introduction and theorem.** We consider the following initial boundary value problem for the nonlinear Schrödinger equation in an exterior domain $\Omega \subset \boldsymbol{R}^n$, $n \geqq 3$:

$$i\frac{\partial u}{\partial t} = \Delta u + \lambda |u|^p u \qquad \text{in } [0, \infty) \times \Omega, \tag{1.1}$$

$$u(0, x) = u_0(x), \tag{1.2}$$

$$u|_{\partial\Omega} = 0. \tag{1.3}$$

Here λ is a real constant and p is an even integer with $p \geqq 2$. The domain Ω is the exterior of a compact set in $\boldsymbol{R}^n$, $n \geqq 3$, with the smooth boundary $\partial\Omega$. In the present paper we shall prove that Problem (1.1)–(1.3) has a unique global solution for small initial data under a certain assumption on the shape of Ω, which indicates that Ω is "nontrapping" in the sense of Vainberg [2] and Rauch [3].

For the Cauchy problem, namely the case of $\Omega = \boldsymbol{R}^n$, the above problem has been extensively studied. For the exterior problem, however, we know only the work of Brézis and Gallouet [1]. In [1] they treated Problem (1.1)–(1.3) only for the case of $n=2$.

We shall first give some notations. For an open set D in $\boldsymbol{R}^n$, let $H^m(D)$, $H_0^m(D)$, $L^2(D)$, $L^1(D)$ and $C_0^\infty(D)$ denote the standard function spaces. We shall fix $R>0$ such that $\partial\Omega \subset \{x \in \boldsymbol{R}^n ; |x| < R\}$. For any $r \geqq R$, we denote the set $\{x \in \Omega ; |x| < r\}$ by Ω_r. We shall often abbreviate $\left(\frac{\partial}{\partial x}\right)^\alpha$ and $\left(\frac{\partial}{\partial t}\right)^j$ to ∂_x^α and ∂_t^j respectively, where α is a multi-index and j is a nonnegative integer. For $a \in \boldsymbol{R}^1$ we denote by $[a]$ the greatest integer that is not larger than a.

Let $G = G(t, x, x_0)$ be the Green function for the following problem:

$$(\partial^2/\partial t^2 - \Delta_x)G = 0 \qquad \text{in } (0, \infty) \times \Omega,$$

$$\lim_{t \to +0} \frac{\partial^j G}{\partial t^j} = \begin{cases} 0, & j=0, \\ \delta(x-x_0), & j=1, \end{cases}$$

$$G|_{x \in \partial\Omega} = 0,$$

where x_0 is an arbitrary point of Ω. For any $\psi(x) \in C_0^\infty(\boldsymbol{R}^n)$ we define $f(t, x, x_0)$ by

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