42. *An Explicit Dimension Formula for the Spaces of Generalized Automorphic Forms with Respect to* $Sp(2, Z)$

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Let $\mathfrak{S}_g$ be the Siegel upper half plane of degree g. The real symplectic group $Sp(g, \boldsymbol{R})$ acts on $\mathfrak{S}_g$ as

$$Z \longmapsto M\cdot Z := (AZ+B)(CZ+D)^{-1},$$

for

$$Z\in\mathfrak{S}_g \quad \text{and} \quad M=\begin{pmatrix} A & B \\ C & D \end{pmatrix}\in Sp(g, \boldsymbol{R}).$$

Let Z and M be as above, and put

$$J(M, Z)=CZ+D \qquad (\in GL(g, \boldsymbol{C})).$$

This satisfies the following relation for any M, $M'\in Sp(g, \boldsymbol{R})$ and $Z\in\mathfrak{S}_g$:

$$J(MM', Z)=J(M, M'\cdot Z)J(M', Z),$$

and this is called the *canonical automorphic factor*. Let μ be a holomorphic representation of $GL(g, \boldsymbol{C})$ into $GL(r, \boldsymbol{C})$. Then $\mu(J(M, Z)) =\mu(CZ+D)$ also satisfies the above relation.

Let μ be as above and let Γ be a subgroup of finite index of $Sp(g, \boldsymbol{Z})$. By an *automorphic form of type* μ with respect to Γ, we mean a holomorphic mapping f of $\mathfrak{S}_g$ to the r dimensional complex vector space $\boldsymbol{C}^r$ which satisfies the equalities:

$$f(M\cdot Z)=\mu(CZ+D)f(Z),$$

for any $M\in\Gamma$ and $Z\in\mathfrak{S}_g$ (we need to assume the holomorphy of f at "cusps" if $g=1$). An automorphic form of type μ with respect to Γ is called a *cusp form*, if it belongs to the kernel of Φ-operator ([1] Exposé 8). We denote by $A_\mu(\Gamma)$ and $S_\mu(\Gamma)$ the spaces of automorphic forms and cusp forms of type μ with respect to Γ, respectively. They are finite dimensional vector spaces. In case $\mu(CZ+D)=\det(CZ+D)^k$, an automorphic form of type μ is also called an automorphic form of *weight* k, and $A_\mu(\Gamma)$ is also denoted by $A_k(\Gamma)$. Similarly $S_\mu(\Gamma)$ is also denoted by $S_k(\Gamma)$.

Let Γ be as above. Then it is known that Γ contains the principal congruence subgroup $\Gamma_g(l)$ of $Sp(g, \boldsymbol{Z})$ for some l, if $g\geq 2$. We may assume that $l\geq 3$. Then $\Gamma_g(l)$ has no torsion elements and the quotient space $\mathfrak{S}_g^*(l):=\Gamma_g(l)\backslash\mathfrak{S}_g$ is non-singular. In the case of degree two the author calculated the dimension of $S_k(\Gamma)$ and represented it by the