5. *Singular Support of the Scattering Kernel for the Wave Equation Perturbed in a Bounded Domain*

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Introduction. Majda [4] obtained a representation of the scattering kernel $S(s, \theta, \omega)$ for the scattering by an obstacle $\mathcal{O}$ (in $\boldsymbol{R}^3$), and showed

(0.1) (i) $\operatorname{supp} S(\cdot, -\omega, \omega) \subset (-\infty, -2r(\omega)]$,
(ii) $S(s, -\omega, \omega)$ is singular (not C^∞) at $s=-2r(\omega)$,

where $r(\omega)=\inf_{x \in \mathcal{O}} x\omega$. In the present note we shall consider the corresponding problems for the acoustic scattering by an inhomogeneous fluid.

Let $a_{ij}(x)=a_{ji}(x) \in C^\infty(\boldsymbol{R}^n)$ $(i, j=1, \cdots, n\ (n \geqq 2))$ satisfy

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geqq \delta|\xi|^2, \qquad \xi \in \boldsymbol{R}^n,$$
$$a_{ii}(x)=1,\ a_{ij}(x)=0\ (i \neq j) \qquad \text{for } |x| \geqq r_0,$$

and set

$$Au=\sum_{i,j=1}^{n} \partial_{x_i}(a_{ij}(x)\partial_{x_j}u).$$

We consider the Cauchy problem

$$\begin{cases}(\partial_t^2-A)u(t, x)=0 & \text{in } \boldsymbol{R}^1 \times \boldsymbol{R}^n,\\ u(0, x)=f_1(x),\ \partial_t u(0, x)=f_2(x) & \text{on } \boldsymbol{R}^n.\end{cases}$$

In the same way as Lax and Phillips [1], [2], we define the scattering operator $S: L^2(\boldsymbol{R}^1 \times S^{n-1}) \to L^2(\boldsymbol{R}^1 \times S^{n-1})$ by $S=T_0^+(W^+)^{-1}W^-(T_0^-)^{-1}$, where T_0^+ (T_0^-) is the outgoing (incoming) translation representation associated with the unperturbed equation and $W^\pm$ are the wave operators (cf. Lax and Phillips [1], [2], the author [6]). S is represented with the distribution kernel $S(s, \theta, \omega)$ (called the scattering kernel) (cf. Majda [4], Lax and Phillips [3], the author [6]):

$$Sk(s, \theta)=\iint S(s-t, \theta, \omega)k(t, \omega)dtd\omega.$$

Let $v(t, x; \omega)$ $(\omega \in S^{n-1})$ be the solution of the equation

$$\begin{cases}(\partial_t^2-A)v=-2^{-1}(2\pi i)^{1-n}(\partial_t^2-A)\delta(t-x\omega) & \text{in } \boldsymbol{R}^1 \times \boldsymbol{R}^n,\\ v=0 \quad \text{for } t<-r_0.\end{cases}$$

$v(t, x; \omega)$ is a C^∞ function of x and ω with the value $\mathcal{S}'(\boldsymbol{R}_t^1)$.

Theorem 1. *Set*

$$S_0(s, \theta, \omega)=\int_{\boldsymbol{R}^n} (\partial_t^{n-2}\Box v)(x\theta-s, x; \omega)dx \quad (\Box=\partial_t^2-\Delta),$$