35. Singular Cauchy Problems for Second Order Partial Differential Operators with Non-Involutory Characteristics

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We denote by (x, y) the variables of C^{n+1}, where $x\in C$ and $y=(y_1, y')\in C\times C^{n-1}$, and by (ξ, η) the dual variables of (x, y). We consider partial differential operators written in the following form:

$$P(x, y, \partial/\partial x, \partial/\partial y)=\sum_{i+|\alpha|\leqslant 2} x^{\kappa(i,\alpha)}a_{i\alpha}(x, y)(\partial/\partial x)^i(\partial/\partial y)^\alpha.$$

Here $\kappa(i, \alpha)$, $i+|\alpha|\leqslant 2$, are integers defined by

$$\kappa(i, \alpha)=\begin{cases} q|\alpha| & i+|\alpha|=2 \\ q' & i=0,\ |\alpha|=1 \\ 0 & \text{otherwise}, \end{cases}$$

where q and q' are integers which satisfy $0\leqslant q'\leqslant q-2$. Furthermore, $a_{i\alpha}(x, y)$, $i+|\alpha|\leqslant 2$, are holomorphic at the origin, and $a_{2,0}=1$.

Remark. If $q'=q-1$, the above operators are said to satisfy Levi condition. Several authors considered singular Cauchy problems for such operators (see Nakane [1], Takasaki [2], and Urabe [4]). Perhaps we can also treat this case, but this requires some modifications which are not trivial. Thus we consider only the case of $q'\leqslant q-2$.

We assume that the equation

$$\sum_{i+|\alpha|=2} x^{q|\alpha|}a_{i\alpha}(x, y)\xi^i\eta^\alpha=0$$

has two roots $\xi=x^q\lambda_i(x, y, \eta)$, $i=1, 2$, where $\lambda_i(x, y, \eta)$, $i=1, 2$, are holomorphic at $x=0$, $y=0$, $\eta=(1, 0, \cdots, 0)$ and homogeneous of degree 1 in η. Furthermore we assume that

$$\lambda_1(x, y, \eta)\neq\lambda_2(x, y, \eta)$$

at $x=0$, $y=0$, $\eta=(1, 0, \cdots, 0)$.

Our purpose is to solve the following singular Cauchy problems:

$$(1)\qquad \begin{cases} Pu(x, y)=0 \\ (\partial/\partial x)^i u(0, y)=\mathring{u}_i(y) \qquad i=0, 1. \end{cases}$$

Here $\mathring{u}_i(y)$, $i=0, 1$, are multivalued holomorphic functions defined on $\{y\in C^n;\ |y_j|<R,\ j=1, 2, \cdots, n,\ y_1\neq 0\}$ with some $R>0$, and satisfy

$$|\mathring{u}_i(y)|\leqslant C\exp\{C|y_1|^{-(q-1-q')/(q+1)}\}$$

with some $C>0$ there.

Let us define $\varphi_i(x, y)$, $i=1, 2$, by