27. On the Structure of Cohomology Groups attached to the Integral of Certain Many-Valued Analytic Functions

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0. Introduction. The present note is a brief summary of our forthcoming paper [7].

Let P_j $(1 \leq j \leq m)$ be non-zero polynomials in n complex variables $z=(z_1, \cdots, z_n)$ and A_j $(1 \leq j \leq m)$ be linear mappings of a finite dimensional complex vector space V. We consider the connection form $\omega=\sum_{j=1}^m A_j(dP_j/P_j)$ *which satisfies the integrability condition* $\omega \wedge \omega=0$. Let D_j be the divisor of $\mathbf{C}^n$ defined by P_j for $1 \leq j \leq m$ and D be the divisor defined by the product $P=P_1 \cdots P_m$. We denote by $\Omega^p_{X^{an}}$ the sheaf of germs of holomorphic p-forms on the complex manifold $X=\mathbf{C}^n-D$. Then the 1-form ω determines an integrable connection ∇_ω on $\Omega^{\cdot}_{X^{an}} \otimes V$ as follows:

$$\nabla_\omega \varphi := d\varphi + \omega \wedge \varphi$$

for each local section φ of $\Omega^p_{X^{an}} \otimes V$. We denote by $\mathcal{S}_\omega$ the complex local system on X defined by the local horizontal sections of ∇_ω. Let $\Omega^p(*D)$ be the set of *rational* p-forms which are holomorphic on X; then we denote by $(\Omega^{\cdot}(*D) \otimes V, \nabla_\omega)$ the complex

$$0 \longrightarrow \Omega^0(*D) \otimes V \xrightarrow{\nabla_\omega} \Omega^1(*D) \otimes V \xrightarrow{\nabla_\omega} \cdots \xrightarrow{\nabla_\omega} \Omega^n(*D) \otimes V \longrightarrow 0.$$

Since X is affine, by the comparison theorem of Grothendieck and Deligne we have isomorphisms

$$H^p(X, \mathcal{S}_\omega) \xrightarrow{\sim} H^p(\Omega^{\cdot}(*D) \otimes V, \nabla_\omega) \quad \text{for} \quad 0 \leq p \leq n.$$

After K. Aomoto, we call the complex $(\Omega^{\cdot}(*D) \otimes V, \nabla_\omega)$ the *twisted rational de Rham complex.*

In the present note, we discuss the vanishing theorems for the twisted rational de Rham cohomology groups $H^p(\Omega^{\cdot}(*D) \otimes V, \nabla_\omega)$ under certain algebraic conditions on the divisors D_j $(1 \leq j \leq m)$ and on the residue matrices A_j $(1 \leq j \leq m)$. This type of studies of cohomology groups of $\mathbf{C}^n-D$ with coefficients in local systems has been made by K. Aomoto from the viewpoint of differential equations ([1]–[4]) and by A. Hattori and T. Kimura from the topological point of view ([5] and [6]). We extend the results of the papers cited above to complex

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