26. *Remarks on the Uniqueness in an Inverse Problem for the Heat Equation. I*

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§1. **Introduction.** For $(p, h, H, a) \in C^1[0,1] \times \mathcal{R} \times \mathcal{R} \times L^2(0,1)$, let $(E_{p,h,H,a})$ denote the heat equation

(1.1) $$\frac{\partial u}{\partial t} + \left(p(x) - \frac{\partial^2}{\partial x^2}\right)u = 0 \qquad (0<t<\infty, 0<x<1)$$

with the boundary condition

(1.2) $$\frac{\partial u}{\partial x} - hu_{|x=0} = 0, \quad \frac{\partial u}{\partial x} + Hu_{|x=1} = 0 \quad (0<t<\infty)$$

and with the initial condition

(1.3) $$u_{|t=0} = a(x) \qquad (0<x<1).$$

Let $A_{p,h,H}$ be the realization in $L^2(0,1)$ of the differential operator $p(x) - (\partial^2/\partial x^2)$ with the boundary condition (1.2), and let $\{\lambda_n \mid n=0,1,\cdots\}$ and $\{\phi(\cdot,\lambda_n) \mid n=0,1,\cdots\}$ be the eigenvalues and the eigenfunctions of $A_{p,h,H}$, respectively, the latter being normalized by $\phi(0,\lambda_n)=1$ $(n=0, 1,\cdots)$. Noting that each λ_n $(n=0,1,\cdots)$ is simple, we call

$$N = \sharp\{\lambda_n \mid (a, \phi(\cdot,\lambda_n))=0\}$$

the "degenerate number" of $a \in L^2(0,1)$ with respect to $A_{p,h,H}$, where $(\ ,\ )$ means the L^2-inner product.

Let T_1, T_2 in $0 \leqq T_1 < T_2 < \infty$ be given. For the solution $u=u(t,x)$ of the equation $(E_{p,h,H,a})$, the following theorem was proved by Murayama [1] and Suzuki [4], differently:

Theorem 0. *The equality*

(1.4′) $$v(t,\xi) = u(t,\xi) \qquad (T_1 \leqq t \leqq T_2;\ \xi = 0, 1)$$

implies

(1.5) $$(q, j, J, b) = (p, h, H, a)$$

if and only if $N=0$, *where* $v=v(t,x)$ *is the solution of* $(E_{q,j,J,b})$ $((q,j,J,b) \in C^1[0,1] \times \mathcal{R} \times \mathcal{R} \times L^2(0,1))$.

In the present paper, for $x_0 \in (0,1]$, we consider

(1.4) $$v_x(t,x_0) = u_x(t,x_0), \quad v(t,\xi) = u(t,\xi) \quad (T_1 \leqq t \leqq T_2;\ \xi = 0, x_0)$$

instead of (1.4′), and study

Problem. *Under what condition on* (p,h,H,a), *does* (1.4) *imply* (1.5)?

Namely, we show when

(1.6) $$\hat{\mathcal{M}} = \{(p,h,H,a)\}$$

is satisfied, where $\hat{\mathcal{M}} = \{(q,j,J,b) \mid C^1[0,1] \times \mathcal{R} \times \mathcal{R} \times L^2(0,1) \mid$ (1.4) holds