3. On Hirota's Difference Equations

By Tetsuji MIWA
Research Institute for Mathematical Sciences, Kyoto University

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§1. The aim of this note is to exploit the operator approach [1] to soliton equations in studying the following non linear difference equation proposed by Hirota [2]:

$$\begin{aligned}(1.1)\qquad &\alpha f(\lambda+1,\mu,\nu)f(\lambda-1,\mu,\nu)+\beta f(\lambda,\mu+1,\nu)f(\lambda,\mu-1,\nu)\\ &+\gamma f(\lambda,\mu,\nu+1)f(\lambda,\mu,\nu-1)=0,\end{aligned}$$

where α, β and γ are constants satisfying $\alpha+\beta+\gamma=0$.

Hirota [2] found the difference Lax pair for (1.1), proved the existence of three soliton solutions and gave an ample list of non linear differential and/or difference equations obtained by taking suitable limits of (1.1). Among them is the KP (Kadomtsev-Petviashvili) equation which is written in Hirota's form as follows:

$$(1.2)\qquad (D_1^4+3D_2^2-4D_1D_3)\tau\cdot\tau=0.$$

He also remarked a significant coincidence of the phase shift term in soliton solutions of the equations (1.1) and (1.2).

Here I shall give an explicit transformation which connects the hierarchy of the KP equation and that of Hirota's difference equation.

One of the striking discoveries of Mikio and Yasuko Sato [3] on the former was that it admits the characters of the general linear group as its solutions with

$$(1.3)\qquad x_j=\text{trace}\,\frac{X^j}{j},\quad X\in GL(N)\quad (j=1,2,3,\cdots)$$

as the continuum variables. The transformation tells us that the latter, in a slightly modified form, admits them also as its solutions with the multiplicities of the eigenvalues of X as the discrete variables.

The transformation gives us also an operator solution to Hirota's difference equation in the sense of [1]. It reduces to operator solutions to equations in Hirota's list in the limit. Here I discuss briefly those for the two dimensional Toda lattice.

Finally I show that a similar consideration for the BKP hierarchy [1] leads us to the following discrete version:

$$\begin{aligned}&\alpha f(\lambda+1,\mu,\nu)f(\lambda-1,\mu,\nu)+\beta f(\lambda,\mu+1,\nu)f(\lambda,\mu-1,\nu)\\ (1.4)\qquad &+\gamma f(\lambda,\mu,\nu+1)f(\lambda,\mu,\nu-1)+\delta f(\lambda+1,\mu+1,\nu+1)\\ &\times f(\lambda-1,\mu-1,\nu-1)=0,\end{aligned}$$

where α, β, γ and δ are constants satisfying $\alpha+\beta+\gamma+\delta=0$.

§2. Let us recall the operator solution to the KP hierarchy.