22. A Characterization of the Intersection Form of a Milnor's Fiber for a Function with an Isolated Critical Point

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§1. Introduction and the statements of the main results. Let $f: C^{n+1}, 0 \to C, 0$ be a germ of a holomorphic function at $0 \in C^{n+1}$ with an isolated critical point. Due to Milnor [2], for r and ε sufficiently small with $0 < \varepsilon \ll r \ll 1$, the restriction

$$f: \{x \in C^{n+1}: |x| < r\} \cap \{|f| = \varepsilon\} \longrightarrow \{t \in C: |t| = \varepsilon\}$$

of f defines a fibration whose general fiber F is a bouquet of n-spheres so that the middle homology group $H_n(F, Z)$ is nonvanishing.

Using Poincaré duality $H_n(F, Z) \simeq H^n(F, \partial F, Z)$, one gets an intersection form $\langle\ ,\ \rangle: H_n(F, Z) \times H_n(F, Z) \to Z$, which is symmetric or skew-symmetric according as n is even or odd.

For a computation of the intersection form, we used in [3] the following fact.

Theorem 1. *A complex valued bilinear form B on $H_n(F, Z) \otimes C$ is a constant multiple of the intersection form if B is invariant under the total monodromy group action on $H_n(F, Z)$, except for the case when f at 0 is nondegenerate* (i.e. ordinary double point) *and n is odd. Here the total monodromy group is by definition the image of the fundamental group of the complement of the discriminant loci of a universal unfolding of f.*

Since this fact seems still not generally well-known, we publish it here with a proof separately from [3]. In §2 we give a somewhat abstract lemma characterizing invariant bilinear forms.

§2. The uniqueness lemma for an invariant bilinear form. Let V be a vector space over a field k with ch $k \neq 2$ and let $\langle\ ,\ \rangle: V \times V \to k$ be a k-bilinear form which is either symmetric or skew-symmetric.

Let A be a subset of V. In case $\langle\ ,\ \rangle$ is symmetric, we assume $\langle e, e\rangle = 2$ for all $e \in A$. Let us associate the graph $\Gamma(A)$ to such A as follows. The set of vertices of $\Gamma(A)$ is in a one-to-one correspondence to A so that we identify them. Two vertices e and e' of A are connected by a 1-simplex if and only if $\langle e, e'\rangle \neq 0$.

Let $W(A)$ be the subgroup of $GL(V)$ generated by the set of reflexions σ_e for $e \in A$, where

$$\sigma_e(u) := u - \langle u, e\rangle e \qquad \text{for } u \in V.$$