12. On Hilbert Modular Forms. II

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(Communicated by Shokichi IYANAGA, M. J. A., Jan. 12, 1982)

Introduction. In his paper [5], J. Igusa gave a minimal set of generators over $\boldsymbol{Z}$ of the graded ring of Siegel modular forms of genus two whose Fourier coefficients lie in $\boldsymbol{Z}$. Also, some problems on the finite generation of an algebra of modular forms were discussed by W. L. Baily, Jr. in his recent paper [1]. The author studied the structure of graded $\boldsymbol{Z}[1/2]$-algebra of symmetric Hilbert modular forms for $\boldsymbol{Q}(\sqrt{5})$ in his first paper [6]. The purpose of this second paper is to describe the minimal sets of generators over $\boldsymbol{Z}$ of the graded rings of symmetric Hilbert modular forms with integral Fourier coefficients for real quadratic fields $\boldsymbol{Q}(\sqrt{2})$ and $\boldsymbol{Q}(\sqrt{5})$. The detailed results with their complete proofs will appear elsewhere.

§ 1. Hilbert modular forms for real quadratic fields. Let $\boldsymbol{K}$ be a real quadratic field and let $\mathfrak{o}_K$ denote the ring of integers in $\boldsymbol{K}$. Let $\mathfrak{H}$ denote the upper-half plane and we put $\mathfrak{H}^2=\mathfrak{H}\times\mathfrak{H}$. We denote by $A_C(\Gamma_K)_k$ the set of symmetric Hilbert modular forms of weight k for $\boldsymbol{K}$, where $\Gamma_K=SL(2,\mathfrak{o}_K)$ is the Hilbert modular group. Let $\mathfrak{d}_K$ denote the different of $\boldsymbol{K}$. Then any element in $A_C(\Gamma_K)_k$ has the following Fourier expansion:

$$f(\tau)=\sum_{\nu\in\mathfrak{d}_K^{-1}} a_f(\nu)\exp[2\pi i tr(\nu\tau)],$$

where the sum extends over the elements ν in $\mathfrak{d}_K^{-1}$ which are totally positive or 0. For any subring R of $\boldsymbol{C}$, define in $A_C(\Gamma_K)_k$ the subset

$$A_R(\Gamma_K)_k=\{f(\tau)\in A_C(\Gamma_K)_k \mid a_f(\nu)\in R \text{ for all } \nu\in\mathfrak{d}_K^{-1},\ \nu\gg 0 \text{ or } 0\}.$$

$A_R(\Gamma_K)_k$ is an R-module, and if we put $A_R(\Gamma_K)=\oplus_{k\geqq 0}A_R(\Gamma_K)_k$, then $A_R(\Gamma_K)$ is a graded R-algebra. Next we shall introduce the Eisenstein series $G_k(\tau)$ of weight k for Γ_K. Let $\sim$ denote an equivalence relation in $\mathfrak{o}_K\times\mathfrak{o}_K$ defined as follows:

$$(\alpha,\beta)\sim(\alpha',\beta') \text{ if } \alpha'=\varepsilon'\alpha,\ \beta'=\varepsilon'\beta \text{ for some unit } \varepsilon' \text{ in } K.$$

For any even integer $k\geqq 2$, we define a series $G'_k(\tau)$ on $\mathfrak{H}^2$ as:

$$G'_k(\tau)=\sum_{(\lambda,\mu)\in\mathfrak{o}_K\times\mathfrak{o}_K/\sim} N(\lambda\tau+\mu)^{-k},\qquad \tau\in\mathfrak{H}^2.$$

where the summation runs through a set of representatives (λ,μ) $\neq(0,0)$. It is well known that the series is absolutely convergent and represents a symmetric Hilbert modular form of weight k for $\boldsymbol{K}$.

We normalize $G'_k(\tau)$ as:

$$G_k(\tau)=\zeta_K(k)^{-1}\cdot G'_k(\tau),$$