116. *A Stationary Free Boundary Problem for a Circular Flow with or without Surface Tension**)

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§ 1. In this note we are concerned with a free boundary problem which is a model for a flow around a planet. The problem is stated as follows.

Problem. Given a unit circle Γ in $\boldsymbol{R}^2$, find a closed Jordan curve γ outside Γ and a function V such that

(1.1) $\Delta V=0$ in Ω_γ,

(1.2) $V|_\Gamma=0$, $V|_\gamma=a$,

(1.3) $\frac{1}{2}|\nabla V|^2+Q+\sigma K_\gamma=$ unknown constant on γ,

(1.4) $|\Omega_\gamma|=\omega_0$.

Here Ω_γ is a doubly connected domain between Γ and γ (see Fig. 1). Constants $a>0$, $\omega_0>0$ and $\sigma\geqq 0$ are given. σ is the surface tension coefficient. Q is a given function defined outside Γ. K_γ is the curvature of γ ($K_\gamma>0$ if γ is convex). $|\Omega_\gamma|$ denotes the area of Ω_γ.

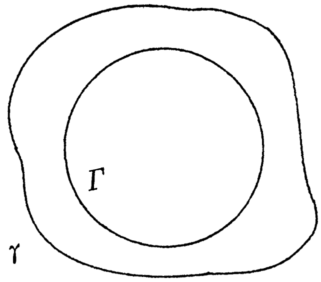

Fig. 1

Remark. We have assumed that the fluid is perfect, irrotational and that V is a stream function for the flow. Ω_γ is the flow region.

The more precise physical meaning of this problem will be explained in a forthcoming paper where we will give proofs of theorems in § 2.

Trivial solution. If Q is radially symmetric, i.e., $Q=Q_0(r)$ ($r=(x^2+y^2)^{1/2}$), then there exists the following trivial solution. Take a number $r_0>1$ satisfying $\pi r_0^2-\pi=\omega_0$. Then a circle γ_0 of radius r_0 with the origin as its center is a solution for any $\sigma\geqq 0$. In fact the

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