114. *Some Dirichlet Series with Coefficients Related to Periods of Automorphic Eigenforms**)

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§1. In this note we construct some Dirichlet series which *generalize* those found in [9, p. 311] and [11, p. 42]. Our basic procedure is to extend the ideas in [9]. Applications will be discussed in a later note.

§2. Let m be any nonnegative integer divisible by 4. Take $R=m/2$. Let q and r be relatively prime, squarefree positive integers. Suppose that:

(2.1) $$y_0^2-ry_1^2-qy_2^2+qry_3^2\neq 0 \qquad \text{for } (y_0, y_1, y_2, y_3)\in \boldsymbol{Z}^4-\{0\}.$$

Cf. [4, pp. 115–116]. Define:

$$S=\begin{pmatrix}1&0&0\\0&-q&0\\0&0&-r\end{pmatrix}\quad S_1=\begin{pmatrix}1&0&0\\0&q&0\\0&0&r\end{pmatrix}\quad S[X]=X^tSX$$

$$n(x)=\begin{pmatrix}1&x\\0&1\end{pmatrix}\quad a(w)=\begin{pmatrix}w&0\\0&w^{-1}\end{pmatrix}\quad k(\theta)=\begin{pmatrix}\cos\theta&-\sin\theta\\ \sin\theta&\cos\theta\end{pmatrix}\quad X=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$\mathcal{M}_z=n(x)a(\sqrt{y})\qquad \text{for } z=x+iy,\quad x\in\boldsymbol{R},\quad y>0$$

$$\mathcal{W}\left[\begin{pmatrix}a&b\\c&d\end{pmatrix}\right]=\begin{bmatrix}\dfrac{a^2+b^2+c^2+d^2}{2} & \sqrt{q}\,(ab+cd) & \sqrt{r}\left(\dfrac{a^2-b^2+c^2-d^2}{2}\right)\\ \dfrac{ac+bd}{\sqrt{q}} & ad+bc & \sqrt{r}\left(\dfrac{ac-bd}{\sqrt{q}}\right)\\ \dfrac{a^2+b^2-c^2-d^2}{2\sqrt{r}} & \sqrt{q}\left(\dfrac{ab-cd}{\sqrt{r}}\right) & \dfrac{a^2-b^2-c^2+d^2}{2}\end{bmatrix}$$

$$\text{for } \begin{pmatrix}a&b\\c&d\end{pmatrix}\in SL(2,\boldsymbol{R})$$

$$\mathcal{V}\left[\begin{pmatrix}a&b\\c&d\end{pmatrix}\right]=\text{the analogous matrix for } S^{-1}$$

$$X_*=\begin{pmatrix}1\\0\\0\end{pmatrix}\quad X_{**}=\begin{pmatrix}0\\1\\0\end{pmatrix}\quad E=k\left(\frac{\pi}{2}\right)\quad \mathcal{D}=\mathcal{W}(E)=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}$$

$$j_Q(z\,;m)=\frac{(cz+d)^m}{|cz+d|^m}\qquad \text{for } Q=\begin{pmatrix}a&b\\c&d\end{pmatrix}\in SL(2,\boldsymbol{R})\quad \text{cf. [5, p. 357].}$$

It is easily seen that $\mathcal{W}$ and $\mathcal{V}$ are *homomorphisms* from $SL(2,\boldsymbol{R})$

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