107. Applications of the Multiplication of the Ito-Wiener Expansions to Limit Theorems

By Gisiro MARUYAMA
Faculty of Science and Engineering, Tokyo Denki University

(Communicated by Kôsaku YOSIDA, M. J. A., Nov. 12, 1982)

We are dealing with a real stationary process

$$X(t)=\sum_{k=1}^{\infty}\int c_k(\lambda)e_k(\lambda,t)d^k\beta, \qquad -\infty<t<\infty,$$
$$e_k(\lambda,t)=\exp(i[\lambda_1+\cdots+\lambda_k]t), \qquad d^k\beta=d\beta(\lambda_1)\cdots d\beta(\lambda_k),$$
$$\lambda=(\lambda_1,\cdots,\lambda_k), \qquad c_k \text{ are symmetric},$$
$$\bar{c}_k(\lambda)=c_k(-\lambda), \qquad c_k\in L^2(d^k\sigma=d\sigma(\lambda_1)\cdots d\sigma(\lambda_k)),$$

where $d\beta$ is the random spectral measure of a real Gaussian stationary process, with $E|d\beta|^2=d\sigma$, which is absolutely continuous $d\sigma(\lambda)=f(\lambda)d\lambda$. We exemplify the multiplication rule through the following simple case.

Let $f, g\in L^2(d^2\sigma)$, then

$$\int f(\lambda,\mu)d^2\beta\int g(\lambda,\mu)d^2\beta$$
$$=\int (f(\lambda,\mu)g(-\lambda,-\mu)+f(\lambda,\mu)g(-\mu,-\lambda))d^2\sigma$$
$$+\int d^2\beta\int\{f(\lambda,\lambda_1)g(-\lambda,\lambda_2)+f(\lambda,\lambda_1)g(\lambda_2,-\lambda)$$
$$+f(\lambda_1,\lambda)g(-\lambda,\lambda_2)+f(\lambda_1,\lambda)g(\lambda_2,-\lambda)\}d\sigma(\lambda)$$
$$+\int f(\lambda_1,\lambda_2)g(\lambda_3,\lambda_4)d^4\beta.$$

Define

$$\Phi(\zeta)=\sum_{m=0}^{\infty}\|c_m\|_2\zeta^m, \qquad \|c_m\|_2^2=\int |c_m|^2\,d^m\sigma,$$
$$M_{2m}=\{\xi\in L^2(\beta):|||\xi|||_{2m}<\infty\},$$

where

$$(|||\xi|||_{2m})^{2m}=\int_0^{\infty}d\mu_m(x)\frac{1}{2\pi}\int_0^{2\pi}|\Phi(\sqrt{mx}e^{i\varphi})|^{2m}\,d\varphi,$$
$$d\mu_m(x)=e^{-x}x^{m-1}dx/(m-1)!, \qquad 1\le m<\infty.$$

Theorem 1. *Suppose we are given $\xi_1,\cdots,\xi_m\in M_{2m}$ and let their IW-expansions be*

$$\xi_i=c_0^i+\sum_{k\ge 1}\int c_k^i(\lambda)d^k\beta, \qquad 1\le i\le m.$$

Multiply the right-hand sides term by term by the multiplication rule as above, and get a formal series of homogeneous polynomials, then