102. *Siegel Modular Forms of Degree Two*

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Introduction. In this note, we discuss a correspondence between the space of modular forms of half integral weight and the space of Siegel modular forms of degree two, and its application to Maass spaces, in close relation with Saito-Kurokawa's conjecture (cf. [2], [3], [4], [8]).

Let M be any positive integer, χ a character mod M, $\tilde{M}=$l.c.m$(4, M)$, and k an even integer. In our previous paper [3], we constructed a linear mapping $\Psi_k^{M,\chi}$ of $\mathfrak{S}_{2k-1}(\tilde{M},\chi)$ into $S_k(\Gamma_0^{(2)}(M),\bar{\chi})$. In this note, we construct another linear mapping Ψ of $\mathfrak{S}_{2k-1}(4N,\chi)$ into $S_k(\Gamma_0^{(2)}(2N),\chi)$, k being an even integer and χ a character mod $2N$. It will be seen that Ψ is more useful than $\Psi_k^{M,\chi}$ in several points and serves to generalize our results in [3]. For example, Theorem 4 in [3] is generalized in the sense that the assumption (5.1) in [3] can be dropped.

§ 1. We denote by $\boldsymbol{Z}$, $\boldsymbol{R}$ and $\boldsymbol{C}$ the ring of rational integers, the field of real numbers and the field of complex numbers. For a ring A, we denote by A_m^n the set of all $n\times m$ matrices with entries in A, and denote A_1^n (resp. A_n^n) by A^n (resp. $M_n(A)$). For a $z\in\boldsymbol{C}$, we set $e[z]=\exp(2\pi iz)$ with $i=\sqrt{-1}$ and we define $\sqrt{z}=z^{1/2}$ so that $-\pi/2<\arg(z^{1/2})\leqq\pi/2$. Denote by $\mathfrak{H}_n$ the complex Siegel upper half space of degree n. Let $Sp(n,\boldsymbol{R})$ be the real symplectic group of degree n. For a positive integer N, we consider a congruence subgroup of the Siegel modular group of degree n defined by

$$\Gamma_0^{(n)}(N)=\left\{\begin{pmatrix}A & B\\ C & D\end{pmatrix}\in M_{2n}(\boldsymbol{Z})\cap Sp(n,\boldsymbol{R})\,|\,C\equiv 0 \pmod N\right\}\quad (\Gamma_0^{(1)}(N)=\Gamma_0(N)).$$

We denote by $S_k(\Gamma_0^{(2)}(M),\psi)$ the space of Siegel modular cusp forms F of Neben-type ψ and of weight k with respect to $\Gamma_0^{(n)}(M)$ satisfying

$$F((AZ+B)(CZ+D)^{-1})=(\bar{\psi}\det(A))(\det(CZ+D))^k F(Z)$$
$$\left(Z\in\mathfrak{H}_n,\ \begin{pmatrix}A & B\\ C & D\end{pmatrix}\in\Gamma_0^{(n)}(M)\right).$$

We consider two symmetric matrices

$$Q_0=\begin{pmatrix}S & 0 & 0\\ 0 & S & 0\\ 0 & 0 & -1\end{pmatrix}\quad\text{and}\quad Q_1=\begin{pmatrix}S & 0\\ 0 & -1\end{pmatrix}\left(S=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}\right).$$

For a positive integer N, set $L(N)=\{{}^t(x_1, x_2, 2Nx_3, (1/N)x_4, \sqrt{2}\,x_5)\,|\,x_i\in\boldsymbol{Z}\}$