101. Multiplier Algebra of C*-Envelope and the C*-Envelope of a Multiplier Algebra*)

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Abstract. Let A be a commutative Banach *-algebra with $C^*(A)$ as its enveloping C^*-algebra. Denote by $M(B)$ the multiplier algebra of a Banach algebra B. The relations between $M(C^*(A))$ and $C^*(M(A))$ are studied in this note. Let $X=\mathcal{M}(C^*(A))$ and $Y=\mathcal{M}(C^*(M(A)))$ be the maximal ideal spaces of $C^*(A)$ and $C^*(M(A))$ respectively. It is proved that if X is dense in Y then $C^*(M(A))$ can be isometrically embedded as a subalgebra in $M(C^*(A))$. If X is not dense in Y, then it is characterized that there is a homomorphism of $C(Y)$ into $C(\beta(X))$ which is induced from the onto map of $\beta(X)$ to $\tilde{X}$ where $\beta(X)$ is the Stone-Čech compactification of X and $\tilde{X}$ is the weak closure of X in Y.

1. Introduction. Let A be a commutative Banach *-algebra with $C^*(A)$ as its enveloping C^*-algebra. Denote by $M(B)$ the multiplier algebra of some Banach algebra B, that is, a subalgebra of bounded linear operators $\mathcal{L}(B)$ of B which commute with algebra product. It is known that the multiplier algebra of a C^*-algebra is also a C^*-algebra. Thus one will know what relations can be established between $M(C^*(A))$ and $C^*(M(A))$. For example

(i) whether $C^*(M(A))\subset M(C^*(A))$?

(ii) what condition can be $C^*(M(A))\cong M(C^*(A))$?

In general we can not say anything about (i) and (ii). But if the character space $X=\mathcal{M}(C^*(A))$ is dense in the character space $Y=\mathcal{M}(C^*(M(A)))$, then certainly (i) holds. While the condition for (ii) is that A is a dense ideal of $C^*(A)$ containing a bounded approximate identity. If X is not dense in Y, then we find only that there is a homomorphism of $C(Y)$ into $C(\beta(X))$, which is induced from the onto map of $\beta(X)$ to $\tilde{X}$ where $\beta(X)$ is the Stone-Čech compactification of X and $\tilde{X}$ is the weak closure of X in Y.

As an example, if G is a locally compact abelian group with dual group $\hat{G}$, then $\hat{G}$ is homeomorphic to the character space $L^1(G)$ as well as the character space of its enveloping C^*-algebra $C^*(G)$ (cf. Bourbaki [2, p. 113]), but $\hat{G}$ is not dense in the character space Δ of the bounded

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