100. *Integral Transforms in Hilbert Spaces*

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1. **Introduction.** We let dm denote a σ finite positive measure and $L_2(dm)$ a usual Hilbert space composed of dm integrable complex valued functions $F(t)$ on a dm measurable set T and with finite norms

$$\|F\|^2_{L_2(dm)}=\int_T |F(t)|^2\, dm(t).$$

For an arbitrary set E and any fixed complex valued function $h(t,p)$ on $T\times E$ satisfying $h(t,p)\in L_2(dm)$ for any fixed $p\in E$, we consider the integral transform of $F\in L_2(dm)$

$$f(p)=\int_T F(t)\overline{h(t,p)}dm(t). \tag{1.1}$$

Then, we first show that the functions $f(p)$ form a Hilbert (possibly finite dimensional) space H which is naturally determined by the integral transform. Furthermore, we establish the fundamental relationship between the two Hilbert spaces $L_2(dm)$ and H.

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2. **The image by the integral transform and norm inequality.** We define the function $K(p,q)$ on $E\times E$

$$K(p,q)=\int_T h(t,q)\overline{h(t,p)}dm(t). \tag{2.1}$$

Note that $K(p,q)$ is a positive matrix on E in the sense of Moore; i.e.,

$$\sum_{\nu=1}^{m}\sum_{\mu=1}^{m}\alpha_\nu\overline{\alpha}_\mu K(p_\nu,p_\mu)\geqq 0$$

for any finite set $\{p_\nu\}$ of E and for any complex numbers $\{\alpha_\nu\}$. This implies that for $K(p,q)$, there exists a uniquely determined Hilbert space H composed of functions on E admitting $K(p,q)$ as a reproducing kernel [2], p. 344 and [1], p. 143. Then, we obtain

Theorem 1.1. *For the integral transform* (1.1), *we obtain*

$$\|f\|^2_H\leqq\int_T |F(t)|^2\, dm(t). \tag{2.2}$$

Further, (1.1) *gives a mapping from* $L_2(dm)$ *onto* H, *and for any* $f\in H$,

$$\|f\|^2_H=\min\int_T |\tilde{F}(t)|^2\, dm(t) \tag{2.3}$$

where the minimum is taken over all functions $\tilde{F}\in L_2(dm)$ *satisfying*