88. *Tensor Products of Singular Holomorphic Representations of SU(n, n) and Mp(n, R)*

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(Communicated by Shokichi IYANAGA, M. J. A., Sept. 13, 1982)

0. Introduction. General theory of tensor products of holomorphic discrete series representations and some of their limits for groups associated with the Hermitian symmetric spaces was established by Jakobsen and Vergne in [3] and Repka in [10]. Some concrete computations of irreducible decomposition of these tensor products are carried out in [3] and [4].

We shall compute in this note the irreducible decomposition of the tensor products of representations "beyond the limits" of holomorphic discrete series of some groups. We restrict our attention only to the group $SU(n,n)$ and the two-fold covering group $Mp(n,\boldsymbol{R})$ of $Sp(n,\boldsymbol{R})$. (See below.) For these groups the irreducible representations of maximal compact subgroups are parametrized by the Young diagrams. Our computation will be reduced to that of Young diagrams. Proofs are done essentially along the line of Jakobsen's proof for a special case in [5]. Details are omitted here.

1. Holomorphic representations of $SU(n,n)$ and $Mp(n,R)$. Let

$$G_1=SU(n,n)=\left\{g\in SL(2n,C)\,;\ g\begin{pmatrix} & -1\\ 1 & \end{pmatrix}g^*=\begin{pmatrix} & -1\\ 1 & \end{pmatrix}\right\}$$

$$G_2'=Sp(n,\boldsymbol{R})=\left\{g\in GL(2n,\boldsymbol{R})\,;\ g\begin{pmatrix} & -1\\ 1 & \end{pmatrix}{}^t g=\begin{pmatrix} & -1\\ 1 & \end{pmatrix}\right\}$$

and G_2 be the metaplectic group $Mp(n,\boldsymbol{R})$, the two-fold covering group of G_2'. Let K_1 and K_2 be the maximal compact subgroups of G_1 and G_2 respectively. The elements of K_1 are complex matrices of the form $\begin{pmatrix} a & -b\\ b & a\end{pmatrix}$ with $((a+\sqrt{-1}b),(a-\sqrt{-1}b))\in S(U(n)\times U(n))$, and those of K_2 are real matrices of the form $\begin{pmatrix} a & -b\\ b & a\end{pmatrix}$ with $a+\sqrt{-1}b\in U(n)$. Let $u=a+\sqrt{-1}b$ and $v=a-\sqrt{-1}b$. We use the unbounded realization D_i of G_i/K_i:

$$D_1=\{z=x+\sqrt{-1}y\,;\ x \text{ and } y \text{ are complex } n\times n \text{ matrices},\ x^*=x,\ y^*=y,\ \text{any } y \text{ is positive definite}\},$$

and

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