79. *Meromorphic Solutions of Some Difference Equations of Higher Order. II*

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1. Introduction. In this note, we will study the difference equation of order n:

(1.1) $$\alpha_n y(x+n)+\alpha_{n-1}y(x+n-1)+\cdots+\alpha_1 y(x+1)=R(y(x)),$$

where $R(w)$ is a rational function of w:

(1.2) $$\begin{cases} R(w)=P(w)/Q(w), \\ P(w)=a_p w^p+\cdots+a_1 w+a_0, \\ Q(w)=b_q w^q+\cdots+b_1 w+b_0, \end{cases}$$

in which $\alpha_n, \cdots, \alpha_1$; $a_p, \cdots, a_0$; $b_q, \cdots, b_0$ are consts, and $\alpha_n a_p b_q \neq 0$. $P(w)$ and $Q(w)$ are supposed to be mutually prime. In the below, we denote by p and q the degrees of the nominator $P(w)$ and of the denominator $Q(w)$, respectively. We put

(1.3) $$q_0=\max(p, q).$$

When $n=1$, the equation (1.1) reduces to

(1.4) $$y(x+1)=R(y(x)).$$

Some properties of meromorphic solutions of (1.4) are studied in [1]–[3]. Especially, we proved in [2, p. 311, Theorem 1], that

(1.5) $\begin{cases}\text{any meromorphic solution of (1.4) is transcendental and} \\ \text{of order } \infty \text{ in the sense of Nevanlinna, if } q_0 \geqq 2.\end{cases}$

(1.5) is not valid if $n>1$, but we proved in [4],

Proposition 1. *When $p>q$, then any meromorphic solution of* (1.1) *is transcendental.*

Proposition 2. *When $p>q+1$, then any meromorphic solution of* (1.1) *is of order ∞ in the sense of Nevanlinna.*

Proposition 3. *When $q_0>n$, then any meromorphic solution of* (1.1) *is transcendental and of order ∞ in the sense of Nevanlinna.*

We will show that Propositions 1–3 are exact, i.e.,

Theorem 1. *Suppose $p \leqq q \leqq n$. Then there is an equation of the form* (1.1) *which admits a rational solution.*

Theorem 2. *Suppose $p=q+1 \leqq n$. Then there is an equation of the form* (1.1) *which admits a transcendental solution of finite order.*

Theorem 3. *Suppose $p \leqq q \leqq n$. Then there is an equation of the form* (1.1) *which admits a transcendental solution of finite order.*

Further, we will show

Theorem 4. *For any p, q, and n, there is an equation of the form*