77. *Semigroups and Boundary Value Problems. II*

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1. Introduction. The purpose of this note is to extend our earlier result [5] on the existence of Feller semigroups to a broader class of degenerate elliptic operators.

Let D be a bounded domain in $\boldsymbol{R}^N$ with smooth boundary ∂D and let $C(\bar{D})$ be the space of real-valued continuous functions on $\bar{D}=D\cup\partial D$. A strongly continuous semigroup $\{T_t\}_{t\geqq 0}$ of bounded linear operators on $C(\bar{D})$ is called a *Feller semigroup* on $\bar{D}$ if $\{T_t\}$ satisfies:

$$f\in C(\bar{D}),\quad 0\leqq f\leqq 1 \quad \text{on} \quad \bar{D}\Longrightarrow 0\leqq T_t f\leqq 1 \quad \text{on} \quad \bar{D}.$$

It is known that there corresponds to a Feller semigroup $\{T_t\}_{t\geqq 0}$ on $\bar{D}$ a strong Markov process $\mathcal{X}$ on $\bar{D}$ and that if the paths of $\mathcal{X}$ are continuous, then the infinitesimal generator $\mathfrak{A}$ of $\{T_t\}$ is described analytically as follows (cf. [1], [6]):

i) Let x be a fixed point of the *interior* D of $\bar{D}$. For a C^2-function u in the domain $\mathcal{D}(\mathfrak{A})$ of $\mathfrak{A}$, we have

$$(1)\qquad \mathfrak{A}u(x)=Au(x)$$
$$\equiv\sum_{i,j=1}^{N} a^{ij}(x)\frac{\partial^2 u}{\partial x_i\partial x_j}(x)+\sum_{i=1}^{N} b^i(x)\frac{\partial u}{\partial x_i}(x)+c(x)u(x)$$

where $(a^{ij}(x))\geqq 0$ and $c(x)\leqq 0$.

ii) Let x' be a fixed (regular) point of the *boundary* ∂D of $\bar{D}$ and choose a local coordinate $x=(x_1, x_2, \cdots, x_{N-1}, x_N)$ as $x\in D$ if $x_N>0$ and $x\in\partial D$ if $x_N=0$. For $u\in\mathcal{D}(\mathfrak{A})\cap C^2(\bar{D})$, we have

$$(2)\qquad Lu(x')\equiv\sum_{i,j=1}^{N-1}\alpha^{ij}(x')\frac{\partial^2 u}{\partial x_i\partial x_j}(x')+\sum_{i=1}^{N-1}\beta^i(x')\frac{\partial u}{\partial x_i}(x')$$
$$+\gamma(x')u(x')+\mu(x')\frac{\partial u}{\partial n}(x')-\delta(x')Au(x')$$
$$=0$$

where $(\alpha^{ij}(x'))\geqq 0$, $\gamma(x')\leqq 0$, $\mu(x')\geqq 0$, $\delta(x')\geqq 0$ and $n=(n_1, n_2, \cdots n_N)$ is the unit interior normal to ∂D at x'. The condition L is called a Ventcel's boundary condition.

In this note we consider the following

Problem. *Conversely, given analytic data* (A, L), *can we construct a Feller semigroup* $\{T_t\}_{t\geqq 0}$ *on* $\bar{D}$?

In [5], the author proved that, under the ellipticity condition on A, if a Markovian particle with generator $L^0=\sum_{i,j=1}^{N-1}\alpha^{ij}\,\partial^2/\partial x_i\partial x_j$ goes through the set $M=\{x'\in\partial D\,;\,\mu(x')=0\}$, where no reflection phenome-